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Determination of Credit Risk for Debt Assets Portfolio
Determinace kreditního rizika u portfolia dluhových aktiv

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2. Description of the Financial Risk
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4. Determination of Credit Risk by Selected Models
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List of Annexes
Annexes

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The Declaration

Herewith I declare that I elaborated the entire thesis, including all annexes, independently.

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1 Introduction

Risk management is very important to reduce the crisis for banks. As we know, banks play a crucial role in the economy by providing payment and credit service. Bank risk management is critical if the banking system have a broad impact on businesses and people. Therefore, all governments regulate the banking. Banks' risks include credit risk, market risk, operational risk, liquidity risk and other risk, among them, credit risk is the most important risk in the bank sector.

The key goal of this thesis is to compute and compare the value of capital requirement for unexpected losses based on credit risk of ten debt assets portfolio under Basle I, Basel II and Basel III and the value of economic capital by using $\text{CreditMetrics}^{\text{TM}}$ model.

There are three main parts of the thesis, description, mathematical analysis and practical part. The description part mainly introduces and describes the several banks' risks, especially the credit risk. In Chapter 2, we introduce some important terms and equations of bank's risks. In mathematical analysis part (Chapter 3), most parts describe the $\text{CreditMetrics}^{\text{TM}}$ model for calculating the economic capital of credit risk. Then, Basel agreements (includes Basel I, Basel II and Basel III) are described in last part of Chapter 3.

For the practical part (Chapter 4), we select ten debt assets with a portfolio traded on Frankfurt Stock Exchange (FSE) to act as the basic data of the thesis. The nominal value of one debt asset is 1 million euro (hence the nominal value of overall portfolio is 10 million euro). The time periods of determining the credit risk is one year. Then is the calculation of the portfolio. First, we use the Basel agreements include the standard approach (SA) and foundation internal ratings-based approach (FIRB) to calculate the value of capital requirement of unexpected losses. Next, the calculation of economic capital by $\text{CreditMetrics}^{\text{TM}}$ model is presented. At the end of the Chapter 4, the conclusion and the comparison of results from the different methods are described.

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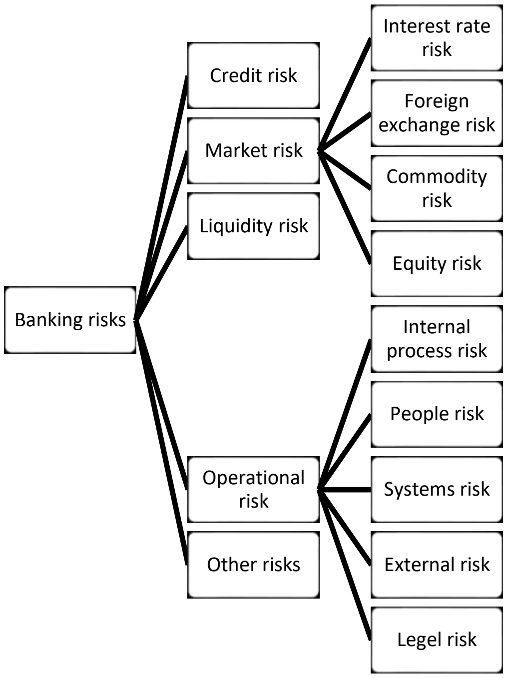
2 Description of the financial risk

In this chapter, the banking risks are introduced. There are four main types of banking risks, credit risk, market risk, operational risk and liquidity risk. Each banking risks would be described respectively. Due to the credit risk is the largest and oldest banking risk in the world, and we will be mainly pay attention to it.

As we know, the important characteristics of bank are stable and trust. When the risk occurs, it means some undesirable things will happen, people, companies or bank will get huge loss from unexpected assets, and some bad effect on investment in the short time, ect. Mentioned on the banking risks, there are some examples, borrowers may not make the payments in time or they do not enough money to pay it, there are some losses on the bank's loans when the changes on market interest rate, and human leads to the error (like fraud in computer system), etc. Therefore, banks are engaged in risk management. It contains some parts, monitoring, managing, and measuring the banking risk. The function of risk management is to manage the risks that the bank faces, continuously measures the risk of current portfolio of assets, transferring the risk, or in good corporation with other bank functions to reduce the possible of loss.

There are five banking risks: credit risk, market risk, operational risk liquidity risk and other risks. Market risk and operational risk have more brunches. The banking risks are described in *Figure 2.1*.

Figure 2.1 Banking risks



2.1 Credit risk

Credit risk (also known as default risk) is the potential loss that the bank would incur if the borrower fails to meet its obligations. The potential loss is also the interest on the loan and repayment of the amount of the loan in accordance with the agreed terms. Most banks faces credit risks and it arise the possibility of the loans or debt held by banks will not repay partially or fully. One example to show the risk-weighted assets (RWA) and capital requirements (CR)in simple way. The *Tab 2.1* shows the two important events RWA and CR of Komerční banka in 2015. The last column is the percentage of each risk-weighted assets divided to total risk-weighted assets.

Tab 2.1 The risk-weighted assets and capital requirements of Komerční banka in 2015 (in CZK million)

	Risk-weighted assets	Capital requirement	%
Credit risk-weighted asset	341,985	25,070	83.89%
Market risk-weighted assets	20,581	186	5.05%
Operational risk-weighted assets	42,270	2,902	10.37%
Additional risk-weighted assets	2,807	1,461	0.69%
Total risk-weighted assets	407,643	29,619	100.00%

Source: Komerční banka annual report (2015)

2.1.1 Types of credit risk

Introducing the types of credit risks is important for well understand the credit risk. There are four types of credit risk that a portfolio of assets or a single asset. These are:

- credit default risk,
- credit spread risk,
- downgrade risk,
- recovery risk.

Credit default risk

Credit default risk is that debtor fails to meet his or her financial obligation. It is also known as default. Another case is technical default which a firm can not make its interest payment on a loan for three months or more, but it has not reach to bankruptcy. All portfolios with credit risk show a credit default risk. The credit risk of an enterprise is measured by its credit rating. The three main credit rating agencies are Moody's, Standard& Poor's (S&P) and Fitch. *Tab 2.2* shows the credit rating of two agencies. It shows the long-term bond in different credit ratings (grad-high rating and speculative rating). These institutions use qualitatively and quantitatively to analyze the borrower, then rate the borrower. There are some problems to be considered in the analysis: the financial position of the firm, for instance, its balance sheet, income statement and cash flow; other specific event of firm such as reformation; the outlook of the industry, etc.

Tab 2.2 Long-term bond credit rating in three rating agencies

Grade-high rating	
Moody's	S&P
Aaa	AAA
Aa1	AA+
Aa2	AA
Aa3	AA-
A1	A+
A2	A
A3	A-
Baa1	BBB+
Baa2	BBB
Baa3	BBB-
Ba1	BB+
Ba2	BB
Ba3	BB-
B1	
B2	B
B3	

Speculative-low rating	
Moody's	S&P
	CCC+
Caa	CCC
Caa	CC
C	C
	CI
	D

Credit spread risk

Credit spread risk is due to the maturity of matching credit-sensitive bonds and risk-free bonds yield variability. It is exhibited by the portfolio from the credit spread which is traded and market-to-market (such as corporate bond, Mortgage - backed securities, etc.). The higher the credit rating is, the smaller the credit spread.

Downgrade risk

Downgrade risk (also known as migration risk) is the risk connected with the deterioration in a conterparty. It also means the when a company or a bond's credit rating declines by the organisation (e.g. rating agency), it often indicates a decline in its credit quality, so investors will demand a higher risk premium.

Recovery risk

Recovery risk indicates that the actual recovery rate is less than the amount originally estimated after the liquidation of the insolvent counterparty assets.

2.1.2 Factors affecting the credit risk

In this subpart, we would introduce which factors will affet the credit risk. As we know, there are many factors affecting the credit risk, such as some macreconomic factors, economic situation, financial regulation, physical situation and so on. However, these factors are not the important for affecting credit risk, among these variables, four of them are important factors:

- probability of default,
- exposure at default,
- loss given default,
- time horizon.

Probability of default

Probability of default, also abbreviated as PD, is the probability that borrowers cn not be able to pay money back on time, usually in one year. In general, the lender estimates that the higher the default probability of the borrower, the higher the interest rate charged by the lender to the borrower (as compensation for higher default risk). If the borrower makes a collateral to the lender, it can reduce the default risk.

In order to evaluate the probability of default, there are some qualitative methods, like some international credit rating agencies such as Moody's, Standard & Poor's, and Fitch. The rating assignment criteria can be divided into two types: internal ratings (assigned by the banks to their consumers) and external ratings (assigned by the agency to bond issuers).

Internal rating system is assigned by the banks to their own customers. It is similar to agency ratings because it also represents the conciseness of a company's ability to fulfill its commitments, based on the assessment of financial risk and business risk. Each bank uses different procedures and practices to rate.

External rating system, also called agency rating system, is assigned by the agency to bond issuers. Agency ratings evaluate the creditworthiness of a borrower and a credit exposure. There are two types of long-term bond in different credit ratings (grad-high rating and speculative rating), we described in previous. In additon to estimate PD, we can use the actuarial approach to calculate the frequency which the company in a given rating class to migrate to other rating classes. It is better to view the rating in one table. *Tab 2.3* shows the transition matrix - Standard & Poor's year-end rating (%).

Tab 2.3 Transition matrix - Standard & Poor's year-end rating(%)

	Year-end rating (%)						
Initial rating	AAA	AA	A	BBB	BB	B	CCC
AAA	88.77	7.80	0.68	0.05	0.10	0.00	0.00
AA	0.68	88.28	7.42	0.55	0.05	0.15	0.02
A	0.07	2.25	87.88	4.88	0.61	0.25	0.01
BBB	0.03	0.28	5.33	83.01	4.44	0.99	0.10
BB	0.02	0.10	0.53	7.07	74.44	7.27	0.79
B	0.00	0.08	0.25	0.41	6.12	73.03	3.32
CCC	0.16	0.00	0.32	0.97	2.26	9.86	53.15

Source: Standard & Poor's (1998)

From *Tab 2.3*, the best rating is that an AAA-rated company has 88.77% probability in one year in the same class. It falls to 53.15% for a company at the initial rating CCC.

Exposure at default

Exposure at default, is abbreviated as EAD, is the amount of expected loss that the bank may face when the debtor defaults. To describe the exposure at default, there are two items should be introduced – expected loss (*EL*) and unexpected loss (*UL*).

Expected loss is also abbreviated as *EL*, is the average of the probability distributions of future losses and it is not the risk. To estimate the *EL* on a credit exposure, the equation is calculated as:

$$EL = EAD \cdot PD \cdot LGD, \tag{2.1}$$

where, *EAD* is the random variables that current risk exposure plus the possible default from now to the loan size; *PD* is the probability of a borrower defaulting; *LGD* is the rate at which a lender suffers a credit risk exposure if the borrower defaults.

Unexpected loss is abbreviated as UL, is the true credit risk (ie, the loss will be greater than the originally estimated risk) is associated with an unexpected loss.

Loss given default

A given default loss rate - or a simple LGD - is the rate at which a lender suffers a credit risk exposure if the borrower defaults. When issuing new loans, LGD is unknowable, even if a default is fully aware, at least if there is no default risk of the secondary market. If, on the contrary, the exposure can be traded in the secondary market, then LGD and RR can be estimated according to the market price after default. Recovery rate is how much you can get back if the client default. It is given by a subtraction recovery rate (RR), and can take any value between 0% and 100%. The equation is as follows:

$$Loss\ given\ defalut = 1 - Recovery\ Rate, \tag{2.2}$$

There are also some factors affecting LGD, and they are grouped into four main types: the characteristics of credit exposure (like collateral)， characteristics of the borrowers (such as the country of borrower, industry), bank's internal factors and other external factors (such as economic cycle of a country, interest rate or inflation).

In order to estimate loss given defalut, the recovery rate should be calculated at first. There are some methods to get recovery rate, such as market LGD, workout LGD and some selected empirical studies. In our thesis, we choose one of the most widely used method is based on seniority market ranking of debt. The recovery rate of defaulted bond is shown in *Tab 2.4*.

Tab 2.4 Recovery rate of defaulteed bond

	Carth & Lieberman			Altman & Kishore		
Seniority class	Number	Mean	Std.dev	Number	Mean	Std.dev
Senior Secured	115	53.80%	26.80%	85	57.89%	22.99%
Senior Unsecured	278	51.13%	25.45%	221	47.65%	26.71%
Senior Subordinated	196	38.52%	23.81%	177	34.38%	25.08%
Subordinated	226	32.74%	20.18%	214	31.34%	22.42%
Junior Subordinated	9	17.09%	10.90%	-	-	-

Source: Carty L.V. e Lieberman D. (1996a)；Altman E. e Kishore, V.M.(1996).

In *Tab 2.4*, it shows more details on studies. Different recovery rate is in different seniority class. High standard deviation shows the different recovery rates of bonds with different seniority and subordination. The standard deviation is always higher than 20%. For example, the average value of recovery rate of senior secured is 53.8%, and the standard deviation is 26.8%.

Time horizon

The key decision to be made establishing credit risk is to choose a time horizon. The time horizon chosen should not be shorter than the timeframe for mitigation actions. There are two types of time horizon: a constant time horizon and hold-to-maturity or run-off time horizon. A constant time horizon which is suitable for excuting in one year. Hold-to-maturity, is also called run-off time horizon, which is usually used in portfolio management.

2.1.3 Ratio indicators of credit risk

Following are the most common and widely used credit risk ratio indicators:

Non performing loans (NPL) is the total amount of borrowings that the borrower has not paid for at least 90 days on a regular basis. Non-performing loans are in default or near default. Once the loan is not fulfilled, the probability that it will be fully repaid is considered to be quite low. The non-performing loan ratio, is the ratio of the amount of nonperforming loans in the bank's portfolio to the total amount of outstanding loans held by the bank. The non-performing loan ratio measures the effectiveness of the bank's repayment of the loan. NPL ratio is calculated by:

$$NPL\ ratio = \frac{Non\ performing\ loans}{Total\ loans\ and\ leases}, \tag{2.3}$$

where, total loans and leases means the outstanding loans in the bank portfolio.

Coverage ratio measure the ability of banks to absorb potential losses from nonperforming loans. The higher the coverage, the greater the bank‘s ability to fulfill its obligations to lenders. The formula is shown as:

$$Coverage\ ratio = \frac{Allowance\ for\ loan\ losses}{Non\ performing\ loans}, \tag{2.4}$$

where, allowance for loan losses are financial reserves established by financial institutions based on estimated credit risk in institutional assets.

Charge-offs ratio is the ratio of net chare-offs loans to total loans and leases.The charge-offs rates are on an annual basis and deducted from recycling. The net charge-offs for the accounting period is equal to the amount of the loan deducted during the period less the recovery, which is the partial or total payment by the customer for the loan that has been offset in the previous accounting period. The equation is as follows:

$$Charge\text{-}off\ ratio = \frac{Net\ chare\text{-}offs\ loans}{Total\ loans\ and\ leases}, \tag{2.5}$$

Loan loss allowance ratio (LLA) is the ratio of allowance for loan losses to total loans and leases or relative to equity capital. Loan loss allowance is provisions for uncollected loans and loans. This provision is used to cover a number of factors related to potential loan losses, including nonperforming loans, customer defaults and renegotiated loan conditions that are lower than previously estimated payments. *LLA* can be computed:

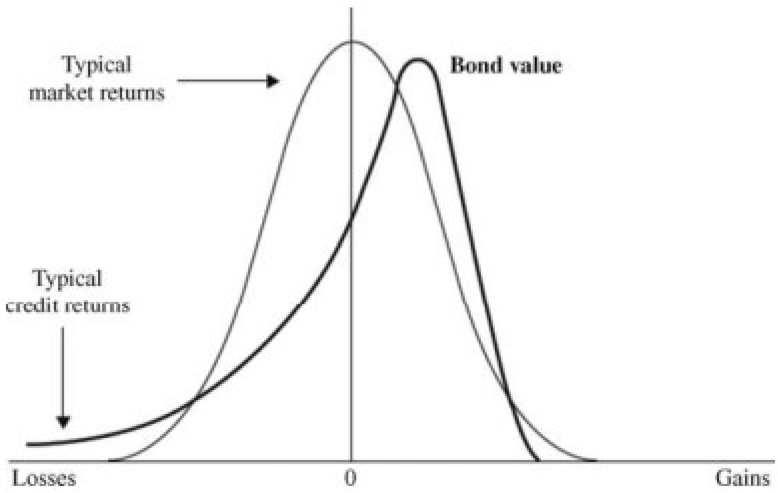
$$LLA\ ratio = \frac{Allowance\ for\ loan\ losses}{Total\ loans\ and\ leases\ or\ relative\ to\ equity\ capital}, \tag{2.6}$$

2.1.4 Difference between credit risk and market risk

Credit risk is different from market risk in nature. In general, the market return distribution is assumed to be relatively symmetric and approximated by a normal distribution.

In the credit portfolio, the value of the change is relatively small due to up or down smally,but the default may be great. The remote probability of such a large loss produces a skew distribution that has a large lower tail portion that is different from the return of normal distribution of the market VaR model. *Figure 2.2* shows the difference distribution of market return and credit return.

Figure 2.2 The difference distribution of market return and crdit return



Source: John Cummins, and Ian Plenderleith. [i]The Principle of Banking. [i] Singapore: John Wiley & Sons Singapore Pte. Ltd., 2012. 914p. ISBN 978–0–470–82521–1.

The market VaR model method mainly considers the time horizon and the VaR of the estimated market distribution. There are two risk measures when models the credit risk: loss distribution: obtain the distribution of losses that may be incurred by the current portfolio and it considers the expected loss at a given confidence level; identifying extreme or catastrophic outcomes: it is resolved by using scenario analysis and concentration limits.

2.2 Market risk

Market risk is the risk that the market price fluctuates as a result of changes in interest rates, exchange rates, stock and commodity prices. Market risk has two components: the impact of similar financial markets, general risk assets or financial markets and only affect the personal financial risk of specific assets. General market risk (also known as systemic risk) refers to the risk market for unfavorable movements that applies to a series of assets. A specific risk is a risk that, due to only applicable factors, the adverse change in the price of a single asset is the safety or issuer and is not related to the general movement of the market.

2.2.1 Types of market risks

There are components of market risks:

- euqity risk,
- interest risk,
- foreign exchange risk,
- commodity risk.

Equity risk

Equity risk is a potential loss due to unfavorable changes in stock prices. A stock, also known as a stock or equity, represents the owner of a company. Banks can purchase ownership of other companies, exposing them to bear the risk of changes in the value of these shares.

Interest risk

Interest risk is the most important market risk. Interest risk is a potential loss due to interest rate changes. This risk arises because bank assets (loans and bonds) generally have a longer maturity than bank liabilities (deposits). This risk can be conceptualized in two ways. First, if interest rates rise, the value of long-term assets will tend to decline, more than the value of short-term liabilities, reducing the interests of banks. Second, if interest rates rise, banks will be forced to pay higher interest rates before their long-term loans expire and can replace them with higher-interest loans.

Foreign exchange risk

Foreign exchange risk (also named as currency risk) refers to the change in the currency exchange rate fluctuation caused by the risk of the value of bank assets or liabilities. Banks trading foreign exchange on behalf of their clients or the bank's own account. Foreign exchange risk can be transaction risk, it can be the translation risk. When the exchange rate changes unfavorable, it generates transaction risk, because foreign currency transactions, you can use different technology hedges. The other risk of conversion is due to accounting risks arising from the translation of assets held in foreign currency or abroad.

Commodity risk

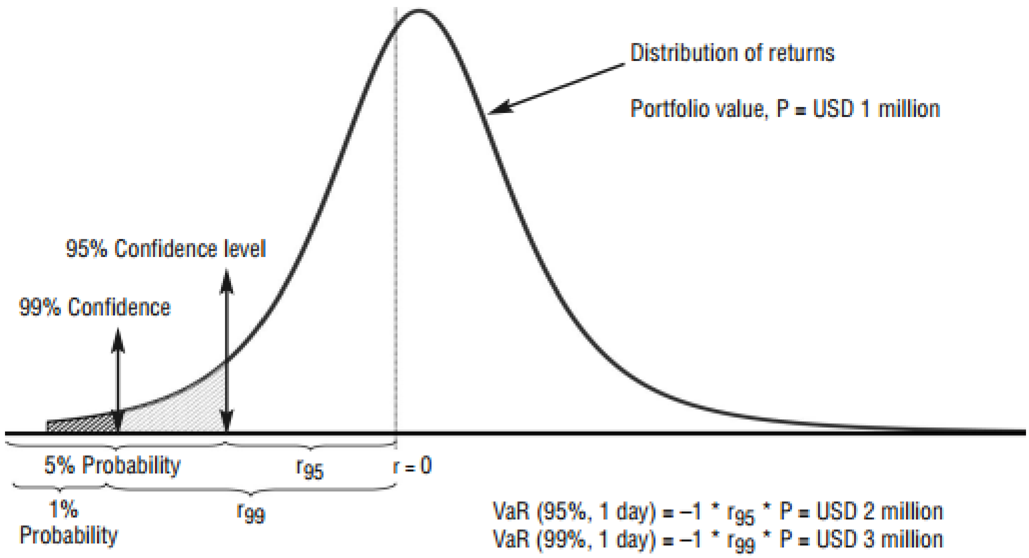
Commodity risk refers to the uncertain future market value and future income scale caused by commodity price fluctuation. There are different types of commodities, including agricultural commodities (such as wheat, soy), industrial commodities (such as metals) and energy commodities (such as natural gas). The value of commodities fluctuates greatly due to changes in demand and supply. Using commodity price hedging strategy is one of the methods that can prevent commodity price uncertainty.

2.2.1 Value-at Risk

In order to measure market risk in their portfolios, banks often use a concept known as value risk. Formally, VaR is defined as a predicted loss for a given time period (e.g., 1 day) at a particular confidence level (e.g., 95% or higher).

Calculating VaR involves carefully examining the current position and estimating the distribution of possible returns over the next period of time (usually a one-day market risk). *Figure 2.3* shows a sample return distribution for a portfolio.

Figure 2.3 A sample return distribution for a portfolio



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 170p.

where, the horizontal X axis represents possible gains and losses. The loss will point to the left of the zero and profit to the right. The area under the curve must be 1. At any given gain or loss, the height of the curve represents the probability of the gain or loss.

2.2.2 Hedging

Banks and individuals to hedge to reduce or offset the risk. In general, hedging involves a position in a derivative that reflects as much as possible the value of the underlying asset.

Hedging is contrary to speculation. In speculative trading, the bank chooses to calculate the risk, expecting a positive future return. In a hedged transaction, the bank chooses to limit some of its exposures by sacrificing some, if not all, of the likely future returns.

2.3 Operational risk

Operational risk exists in almost all banking transactions and activities and is a major concern of banking regulators, regulators and bank management. Operational risk is a potential loss due to internal processes or system failures or failures, human error or external events, such as fraud and theft.

2.3.1 Types of operational risk

There are five operational risk event according to Basel II:

- interal process risk,
- people risk,
- external risk,
- system risk,
- legal risk.

Interl process risk

Internal process risk is a risk associated with a bank process or program failure. For example, lack of control - bank staffs failed to audit record transactions between banks and customer accounts; transaction error - a teller adds an extra zero to a deposit, for example, making it RMB 2,000 instead of RMB 200; inaccurate or inaccurate reports required by the banking regulator. Reviewing and improving the internal processes of banks to improve operational risk management tends to improve the operational efficiency and overall profitability of banks. Frequent auditing and analysis procedures also can reduce internal process risks.

People risk

People risk, employee error or fraud-related risks are common sources of operational risk. People's risk can occur in every part of the bank even in the bank's risk management function. People risk occurs often. For example, frequent personnel changes mean that the new person has no experience or training; the process may not be fully understood; poor management practices; excessive reliance on key employees leads to burnout at work.

External risk

External risk is a risk associated with an event occurring outside the direct control of the bank. External risk events are often rare, but when they occur, they can have a significant impact on the bank's operations, large enough to cover a wide range of media coverage. Such as external events are mass robbery, fire, natural disasters, riots and civil protests.

System risk

System risks (also known as technology-related system) are related to the use of computer technology and computer systems. There are some cases of system risks. Insufficient project control - failure to correctly plan the quality of the risk reports that may affect the computer system generation. Data corruption - surge changes data when processing data. System security issues - computer viruses and computer hacking.

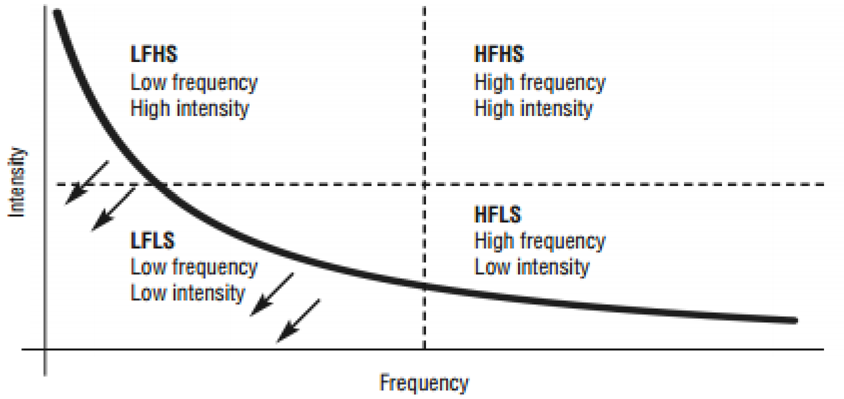
Legal risk

Legal risk is the risk of uncertainty associated with the application or interpretation of legal actions or contracts, laws or regulations. Legal risks vary from country to country; in some cases, unclear laws can lead to murky legal interpretations.

2.3.2 Operational loss events

Operational loss events are typically classified according to the frequency with which they occur and the severity of the potential loss. Operational risk management practices include two general types of losses: loss events that frequently occur with low impact or severity (high frequency / low impact events); and infrequent but high impact (low frequency / high impact events) event. *Figure 2.4* show the loss intensity and frequency chart of operation risk events.

Figure 2.4 Loss intensity and frequency chart of operation risk events



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 188p.

Loss of high-frequency / low-impact (HFLI) operational risk may be small, but in general, HFLI events are considered important. Many financial service providers will consider these losses in their product pricing structures. Low-frequency / high-impact (LHFI) operational risk is the loss of very few occurrences and it is difficult to model and predict these events. For instance, rogue traders, terrorist attacks and fires.

2.4 Liquidity risk

Liquidity risk is related to the bank's ability to fulfill its continuing obligations, including the financing of its assets. Solvency differs from the ability of banks to make profits. Banks may be solvent (assets more than liabilities), and if expenses are greater than their income, they can not make a profit. For example, when a bank makes a large loan but the borrower is unable to pay it, the bank's liabilities (deposits and borrowings) may exceed the bank's assets, causing the bank to become insolvent. There are some ratio indicators of liquidity risk.

Liquidity (quick) ratio

Liquidity (quick) ratio is more common and widely used. For banks, this is the proportion of bank holdings in bank deposits. The liquidity ratio measures the degree of rapid liquidation of assets and covers short-term liabilities. The formulas are computed:

$$Liquidity\ ratio = \frac{\begin{array}{c}Cash{+}Cash\ due\ from\ banlance\\ held\ at\ other\ financial\ institution\end{array}}{Total\ assets}, \tag{2.7}$$

or

$$Liquidity\ ratio = \frac{Cash\ assets\ and\ government\ securities}{Total\ assets}. \tag{2.8}$$

Liquidity coverage ratio

Liquidity coverage ratio (LCR) is a high-liquidity asset held by financial institutions to meet short-term liabilities. LCR is calculated by the banks' stock of high-quality liquid assets divided to the total net cash outflow over the 30-day stress period. The formla can be written as follows:

$$Liquidity\ coverage\ ratio = \frac{Stock\ of\ high-quality\ liquid\ assets}{\substack{Total\ net\ cash\ outflow \\ over\ 30\ calender\ days}}, \tag{2.9}$$

where high-quality liquid assets include those which can be easily converted into cash.

Liquidity monitoring - banks need to have sufficient and readily available funds to satisfy regular withdraws and meet their loan obligations.

2.5 Other risk types

Besides these four main financial risks, there are also some important types of risks, including business risk, reputational risk and regulatory.

Business risk

Business risk is a possible loss due to a decline in the competitive position of banks and the prospect of a boom in the changing market. Business risks can be affected by two main risks: internal risk (the risk of an event occurring within the organization) and external risk (the risk of an event occurring outside the organization).

Reputational risk

Reputation risk refers to the loss that an organization's goodwill may face. It can damage the corporate image, force it into costly litigation cases, and cause loss of revenue and loss of customers or key employees. For example, a plane crash, the airline made a rapid response and the degree of moral care will directly affect the company's goodwill on the judge.

Regulatory risk

As the regulatory institution (such as government) will make changes (or impose the new rules) to the current rules, it will have a negative impact on the transactions that have already taken place and result in a financial loss risk.

3 Description of the credit risk management and models

In this chapter, there are three parts. At first, the types of credit risk management are introduced, such as scoring models, rating system and portfolio models. Then we focus on how the CreditMetrics™ works and introduce the regulation of capital requirements Basel Ⅰ, Basel II and BaselIII.

3.1 Models of credit risk management

From several decades, more and more banks need sophisticated systems to make their banks healthy, safety. The world's largest banks developed some sophisticated systems to estimate the credit risk of their internal and external management in order to improve bank's quantify and efficiency. There are three types of model of credit risk management: scoring model, rating system and portfolio models. These models are introduced in each subchapters.

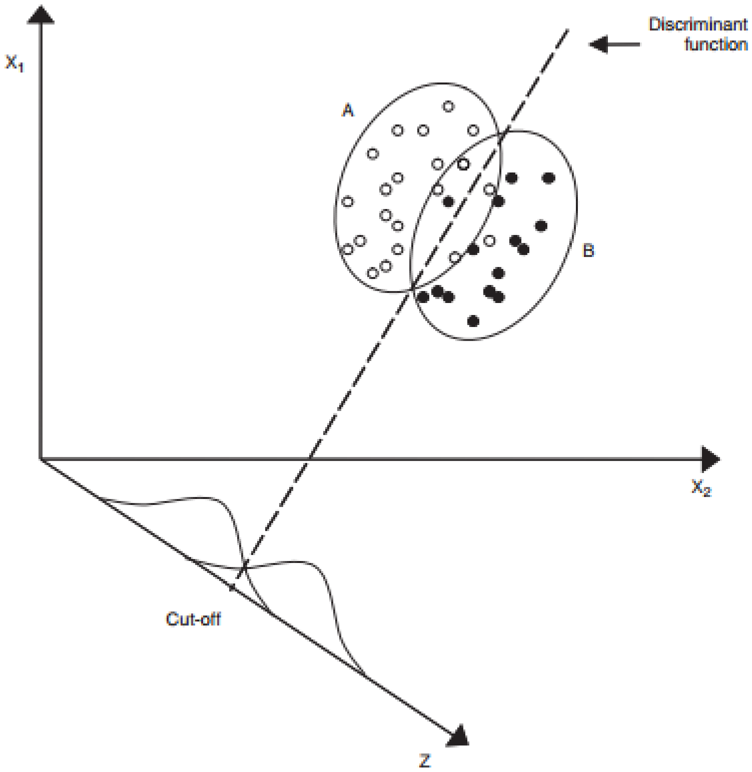
3.1.1 Scoring model – Altman's Z-score

Scoring model is the best-known model to forecast a company's default and it is a type of statistic models. To use this model, some main economic and financial indicators of a company should be the inputs, then assigning weights to each of them, this work can reflect the relative importance of the forecasting default. The result of the scoring model is shown as a numerical score and it indirectly measures the borrower's probability of default.

In this subchapter, we focus on one categories of credit-scoring models—Linear discriminant analysis. Linear discriminant analysis is the most basic model of credit-scoring model which was studied by Fisher as early as 1936. It uses some identification of the variables (such as economic or financial ratio from financial statements) to determine whether the companies is healthy or not.

First, the discriminant analysis should be introduced. Discriminant analysis is a classification technique, which uses the data obtained from the company sample to draw a border that separates a reliable set of groups with insolvency. The Figure 3.1 shows the Fisher model in the (simplified) case in which reliable (A) and insolvent companies (B) are described by only two variables, x_1 and x_2. The score generated by combining of two variables is shown on the z axis.

Figure 3.1 Graphic representation of Fisher model



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 288p.

In the simplest version of discriminant analysis, constructs the score z as a linear combination of the independent variables, x_1 and x_2. The formula is calculated as follows:

$$z = \sum_{j=0}^{n} \gamma_j x_j, \tag{3.1}$$

where, n is independent variables in more general terms.

In the generic, using ith as a company, the score is show as (3.2):

$$z_i = \sum_{j=1}^{n} \gamma_j x_{i,j}, \tag{3.2}$$

where, γ_j is the coefficients of this linear combination which can obtain a score z as clearly as possible between abnormal and healthy companies.

The most popular discriminant score applied to credit risk is Altman's Z-score which developed by Edward Altman in 1968 in US. The function uses five independent variables to calculate, the formula is as follows:

$$z_i = 1.2 \cdot x_{i,1} + 1.4 \cdot x_{i,2} + 3.3 \cdot x_{i,3} + 0.6 \cdot x_{i,4} + 1.0 \cdot x_{i,5}, \tag{3.3}$$

where, x_1 is the working capital / total assets, x_2 is the retained profits / total assets, x_3 is earnings before interest and tax / total assets, x_4 is market value of equity / book value of total liabilities and x_5 is turnover / total assets.

The higher the z score of a company, the better of its quality. Altman sets the cut-off point at 1.81 to determine the relative healthy companies and excessively risky companies. The cut-off value is calculated by the average between the mean value of z form a sample of healthy companies and that from a sample of insolvent companies. If the value is higher than 1.81, the companies are relative healthy; lower than 1.81, the companies are more risk of Altman's model.

3.1.2 Rating system

In this part, rating systems are introduced. Rating systems includes two categories: qualitative methods and quantitative methods. Qualitative methods are used by international credit rating agencies like Moody's, Standard & Poor's and Fitch. Quantitative methods are more likely discriminant analysis or logit, it usually uses to summarize the company's financial indicators. Generally speaking, there are three steps of rating system:

· rating assignment,

· rating quantification,

· rating validation.

· rating assignment

There are two assignment of rating assignment: internal ratings and agency ratings. Internal rating and agency ratings are similar, which represents a concise summary of a company's ability to fulfill its commitments to assess financial risks and business risks. Moreover, some main factors would be used to review, such as profitability, financial leverage, industry, liquidity, etc.

Rating quantification

After rating the borrowers, the latter must be calculated as a probability of default (PD), then it can be used for risk measurement purposes. There are three approaches to solve the problem:

- the statistical approach,
- the mapping approach,

- the actuarial approach.

The statistical approach is usually calculated an individual PD based on the score from a credit-scoring model. The mapping approach is more like a link between the internal ratings and the agency ratings.

From three approaches, we focus on the actuarial approach. The actuarial approach based on actual default frequencies. This approach uses the past default rates recorded in the rating classes to estimate the future PD of all borrowers of each class. For example, If the past data shows that 5% of customers who are assigned to class B tend to default within one year, then 1% of the PD will be assigned to all borrowers now in the class B. There are three rates included in the actuarial approach: marginal rates, cumulative default rates and annualized default rates.

The marginal default rate calculated the sample of bond issues can be used as an estimate of the probability of an insolvency within t years. The formula of marginal default rate for year t is calculated as:

$$d'_t = \frac{D_t}{N_t} \ , \tag{3.4}$$

where D_t represents the number of defaults recorded in year *t*, N_t is the number of issuers present at the beginning of year *t*.

The marginal survival rate can also be used to estimate the probability in year t. The function is shown as follow:

$$s'_t = \frac{N_t - D_t}{N_t} = 1\text{-}d'_t \tag{3.5}$$

The cumulative default rates represents the probability of default for a given number of years after the issuance of the bond. The formula of the cumulative default rates for the period between 0 and T can be written as :

$$d_T = \frac{\sum_{t=1}^{T} D_t}{N_1}, \tag{3.6}$$

The relative cumulative survival rate between 0 and T is shown as:

$$s_T = 1 - p_T = \frac{N_1 - \sum_{t=1}^{T} D_t}{N_t} \ , \tag{3.7}$$

In addition, at the given definition, $N_{t+1} = N_t - D_t$, the s_T and d_T can be also rewritten as:

$$s_T = \prod_{t=1}^{T} s'_t, \tag{3.8}$$

$$d_T = 1 - \prod_{t=1}^{T} (1 - d'_t) = 1 - s_T, \tag{3.9}$$

It can also calculate the corresponding average annual default rate $\overline{d_T}$ from the beginning of a cumulative default rate d_T. The formula is given as:

$$\overline{d_T} = d^* \mid 1 - \prod_{t=1}^{T}(1 - d^*) = 1 - (1 - d^*)^T = d_T, \qquad (3.10)$$

Rating validation

Rating system should be checked regularly to assess its effectiveness. Generally, there are two aspects should be covered in the validation: the quality of the inputs fed into the system and the reliability of the methods used to process them. To assess the performance of the bank rating distribution process, we focus on the simple quantitative criteria (contingency table, ROC curve).

The first method of quantitative criteria is contingency table. The contingency table is a matrix that compares the predictions of the model with the actual events. *Tab 3.1* shows the example of a contingency table.

Tab 3.1 the example of a contingency table

		Performing	*Defaulting*
Rating by model	*Low-risk* (*"pass"*)	Correct valuation (N_1 cases)	Type I errors (N_2 cases)
	High-risk (*"fail"*)	Type II errors (N_3 cases)	Correct evaluations (N_4 cases)

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 389p.

where, N_1 is the number of companies rated as "healthy";

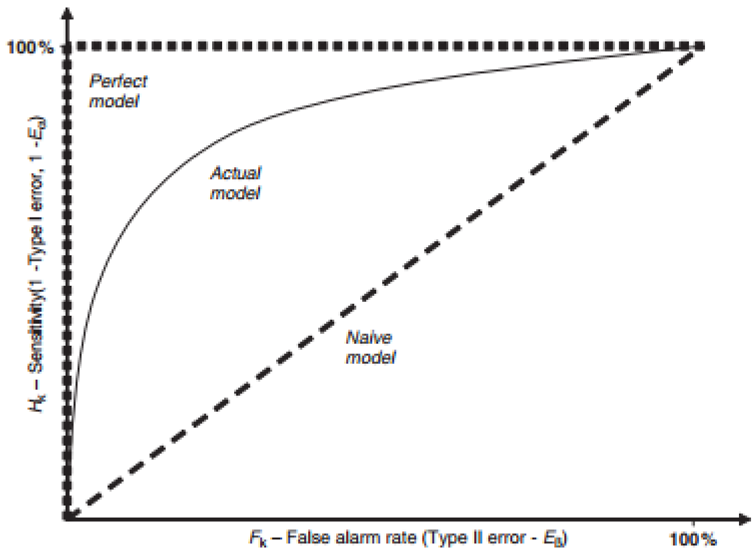
N_2 is the number of companies incorrectly rated as healthy;

N_3 is the number of companies incorrectly rated as being too risky;

N_4 is the number of companies rated as high-risk.

The second method for model validation is the ROC (Receiver Operating Characteristic) curve. This is a graph analyzing the error levels associated with all possible values of the cut-off point that separates “pass” from “fail” borrowers[1]. From the *Figure 3.2* below, the steeper the slope of the initial stretch of the curve, the better of the model.

Figure 3.2 the example of ROC curve



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 391p.

3.1.3 Portfolio models

Portfolio models focus on how to estimate the unexpected loss (such as economic capital). There are two approaches to quantify unexpected loss.

First is the simplest one – standard deviation of the probability distribution of future losses. The second approach is the measurement of value at risk.

Portfolio models seek to determine the maximum losses that a credit portfolio may face in a predetermined time. There are four models to analyze:

- CreditPortfolioView™,
- CreditRisk +™,
- PortfolioManager™ (KMV),

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[1] ANDREA, S. and ANDREA, R. *Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies.* Wiley Finance, 2007. 390p.

• CreditMetrics$^{\text{TM}}$.

There are two basic approach of the credit loss and credit risk:

Market to market: due to the default, the credibility of the asset may change and have a potential impact on the bank's financial position.

Default model: is a model for assessing the likelihood of a borrower's default. The default model can be integrated with a correlation model to model the credit risk of the portfolio. It can use rating historical frequencies of migrations to build the model.

In order to analyze the individual model, there are two problems should be solved:

The choice of the risk horizon : it is important to specify the future intervals we wish to mention because the distribution of losses for the next one year will be more uncertainly than that for the next day.

The choice of the confidence level : different confidence level will lead to different results of value at risk. It is necessary to choose the reasonable confidence level.

CreditPortfolioView$^{\text{TM}}$

CreditPortfolioView$^{\text{TM}}$ was developed by Tom Wilson in 1997. This model is based on the credit cycle of the observation that depends on the economic cycle. Hence, the migration to higher rating class (upgrades) tends to be more frequent during the period of economic growth, while the mobility to lower class (downgrades) and default rates is reduced. It uses the probabilities of migration and default to link with some macroeconomic variables (for example, interest rate, inflation, unemployment rete, ect.) to adjust them to the economic cycle.

Considering the probability of default p_{jt} at time t of a group or segment j of companies which represent the macroeconomic variables with the economic cycle, the logic function can be written as:

$$p_{ij} = \frac{1}{1+ e^{-y_{j,t}}} \, , \tag{3.11}$$

note $y_{j,t}$ indicates the value of a health index at time t of the segment j. The higher the value, the lower the default probability.

CreditRisk $+^{\text{TM}}$

CreditRisk $+^{\text{TM}}$ applies to some typical insurance mathematic instruments for credit risk. It was developed by *Credit Suisse Financial Products* in 1997 (CSFP, 1997). This model can only concentrate on default risk. Although there are some limitations, CreditRisk $+^{\text{TM}}$ is very effective in estimating risk with a large number of positions in the portfolio. Therefore, it is widely used to manage some of the traditional banking portfolios, such as mortgages, small and medium-sized loan enterprises, consumer loans, etc.

In this model, we focus on the probability distribution of defaults. CreditRisk $+^{TM}$ describes the probability distribution of the number of future defaults over a one year through a typical tool of actuarial mathematics (that is, Poisson distribution). The probability of n defaults is calculated as:

$$P\ (n) = \frac{e^{-\mu}\mu^{n}}{n!}, \tag{3.12}$$

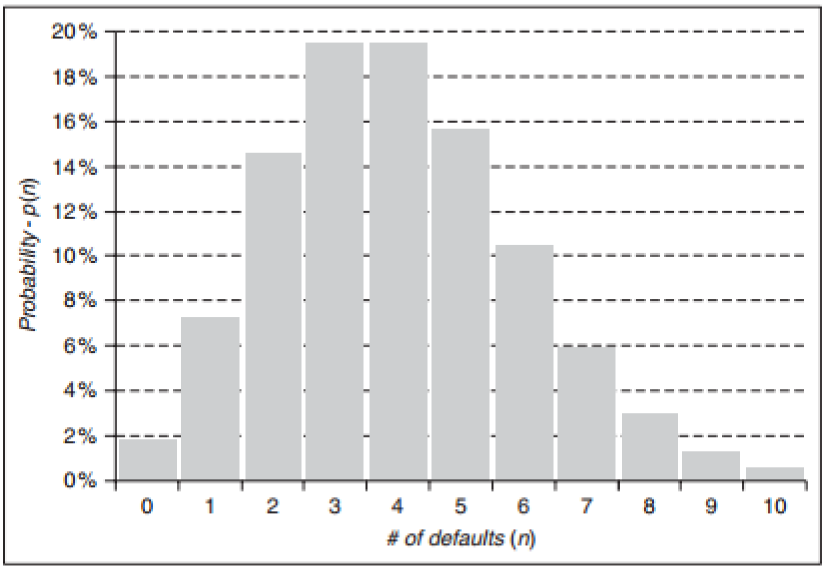
where, μ is the expected number of defaults and indicates the sum of all PDs of the customers in the portfolio.

For instance, considering that a bank has 300 customers, and each with a PD of 1%, the value of μ will be 3. There is no defaults occurs. We use formula (3.12) to calculate the result:

$$P\ (0) = \frac{e^{-3}3^{0}}{0!} = 4.98\ \%$$

Figure 3.3 shows the values of p(n) between 0 and 10 of Poisson distribution. The probability distribution thus obtained is skewed to the right. The value of *n* is higher, the probability would decrease closely to zero.

Figure 3.3 Example of Poisson distribution



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 430p.

PortfolioManager^{TM}

Then, we introduce the PortfolioManager^{TM} (also named KWV) model. PortfolioManager^{TM} model is developed by the California-based company KMV. It is based on Merton model and the method of estimating the default probability. This model claims equity value (E) is equal to the value of a call option to the value of the company's asset market, which the maturity is equal to the residual life debt (T) of the company and the exercise price is equal to the nominal repayment value of the debt (F). *Tab 3.2* shows the different positions produce the same result at maturity T. *Figure 3.4* shows the graphically way of matrix.

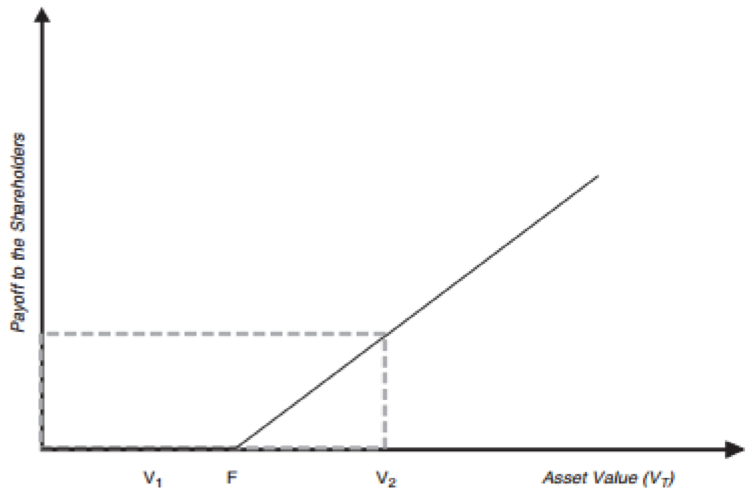
Tab 3.2 Matrix of payoffs as a shareholder or for the purchase of a call option on asset value with a strike price of F.

	Payoff at time 0	Payoff at T	
		if $V_T < F$	if $V_T > F$
Shareholder	$-E_0$	0	$(V_T - F)$
Purchase of a call option	$-C_0$	0	$(V_T - F)$

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 332p.

From the *Tab 3.2*, we can see, if the V_T is lower than face value of debt, the company can not pay back the money and the remaining assets are used exclusively for debt repayment. Therefore, shareholders lose their initial investment and receive nothing. Conversely, when the V_T is higher than face value of debt, the difference of V_T – F is the wealth of shareholders. *Figure 3.4* shows the graphically way of matrix.

Figure 3.4 Shareholder payoff profile



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 330p.

There are some steps to estimate the probability of default of company by using the KMV model.

Default point (*DP*) is equal to all short-term debt (*STD*) plus 50% of long-term debt (*LTD*). The formula can be written as:

$$DP = STD + \frac{1}{2} LTD, \tag{3.13}$$

Then we should calculate the distance to default (*DD*). The function of distance to default is shown as follow:

$$DD = \frac{V_0 - DP}{V_0 \cdot \sigma_V}, \tag{3.14}$$

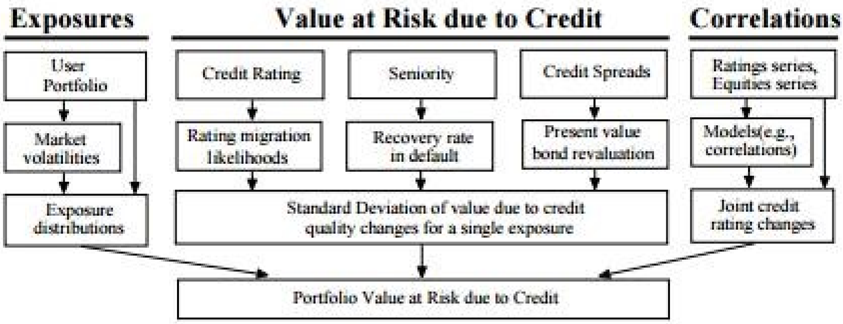
where, V_0 is the value of asset, *DP* is the default point, σ_V represents a multiple of the standard deviation of assets.

Once knowing *DD*, we can use a fairly precise empirical correlation to calculate the probability of default, which also refers to as expected default frequency (EDF).

3.2 Description of CreditMetrics^{TM}

CreditMetrics^{TM} is one of the best-known model to forecast the credit risk on a portfolio of exposures. It is introduced by the US bank J .P .Morgan. CreditMetrics^{TM} is a method of estimating the distribution of change of market value of the credit risk portfolio, usually occurs at a given time horizon (one year). It is possible to estimate the expected loss (EL) and unexpected loss (UL) based on this distribution. *Figure 3.5* shows the framework of step-by-step introduction of CreditMetrics^{TM} model.

Figure 3.5 Framework of $CreditMetrics^{TM}$ model



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. iv.

In this figure, we can see there are three main parts of $\text{CreditMetrics}^{\text{TM}}$ model, including exposures, value at risk dur to credit and correlations. In the subchapter, we will describe risk measurement framework, model parameters, and interpretation and applications.

VaR (Value at Risk) states the risk level, and it describes a minimal expected loss on a given probability (usually 99%, 99.5% or higher) in a specified time period. There are two ways to define the VaR:

At the stated significance α, the loss of the portfolio random profit ($-\Delta\widetilde{\Pi}$) should be higher than predetermined level VaR. The equation can be expressed as follows:

$$Pr(-\Delta\widetilde{\Pi} \geq VaR) = \alpha, \tag{3.15}$$

The loss of the portfolio random profit should be lower than predetermined level $-VaR$, if the significance is α. The equation can be written as follows:

$$Pr(\Delta\widetilde{\Pi} \leq -VaR) = \alpha, \tag{3.16}$$

VaR also can be extended to the Monte Carlo method. This model is based on a large number of simulations which generate the development of the portfolio assets. Then, the model is used for determining the probability distribution if the value of the portfolio assets ($\Delta\widetilde{\Pi}$) increase with the stated significance level α. The formula is shown as follows:

$$\Delta\widetilde{\Pi} = \widetilde{V}_P^T - V_P^t = \sum_n \widetilde{V}_{n,i,T} \cdot X_n - \sum_n V_{n,i,t} \cdot X_n, \tag{3.17}$$

note $\widetilde{V}_P^T$ denotes the default value of the portfolio, V_P^t is the forecast value of the portfolio, $V_{n,i,t}$ represents the value of the *n-th* asset with *i-th* rating class at the end of a given maturity T, X_n indicates the amount of *n-th* asset with *i-th* rating class, $V_{n,i,t}$ is the value of n-th asset within *i-th* rating class in the portfolio.

Economic capital (EC) is the difference between expected loss and unexpected loss. It defines the amount of capital estimated by the bank to maintain the solvent at a given level of confidence and time. The formula generates by VaR can be expressed as follows:

$$EC = VaR_\alpha - E(-\Delta \widetilde{\Pi}), \tag{3.18}$$

note VaR_α denotes the simulated values of the portfolio returns by the combination are arranged in order and the value of VaR at the stated probability will be equal to *n-th* worst, $E(-\Delta \widetilde{\Pi})$ represents the mean value of gains.

3.2.1 Risk measurement framework

There are three steps to calculate the credit risk for a portfolio of one bond as illustrated in Figure 3.5 above:

· step 1: Credit rating migration;

· step 2: Valuation of a bond;

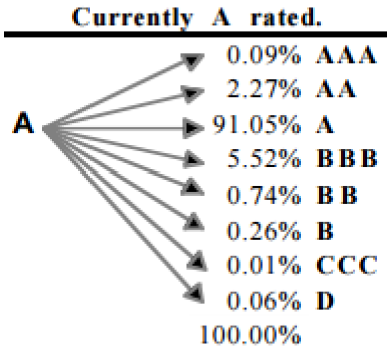
· step 3: Valuation of discount rate;

· step 4: Credit risk estimation.

Step 1: Credit rating migration

The risk comes not only from the default, but also from the value of the change (below) level. Therefore, it is important to calculate the likelihood of default and the chance of migration to any possible credit quality at a given time. The likelihood of any credit rating migration in the next period depends on the debtor's senior unsecured credit rating. There is an example shows the credit quality migration likelihoods for debtors currently rated A in one year (*Figure 3.6*).

Figure 3.6 Example of credit quality migrations (in one year)



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 24p.

Figure 3.6 shows, for instance, there is a 2.27% chance that a A rated will upgrade to a AA rated credit within one year. Generally, the most likely credit rating is the currently rating within one year (that is, 91.05% of A rating).

Step 2: Valuation of a bond

In this step, the value of the risk level of these credit quality states will be determined. For one-bond example, each migration state should be calculated once and there are eight revaluations for it. The eight valuations includes two types.

First is the event of a default, it calculates the recovery rate based on the seniority classification of the bond. *Tab 3.3* shows the recovery rates in the state of default.

Tab 3.3 Recovery rates by seniority class (% of face value, I.e., "par")

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Carty & Lieberman [96a] – Moody's Investors Services

From the *Tab 3.3*, we can see the mean recovery rate (%) and standard deviation (%) of each seniority class. For example, BBB bond is senior unsecured, its mean value of default is 51.13% of its face value (we assume the face value is \$100) and the standard deviation of the recovery rate is 25.45%.

Second is the valuation in the states of up or down grade. In order to obtain a value corresponding to the risk level of the rating (lower) level, we perform a direct present value bond revaluation. The function of value of the bond (V) can be written as :

$$V = c + \frac{C}{(1+i)} + \frac{c}{(1+i)^2} + \frac{c}{(1+i)^3} + \ldots + \frac{c+M}{(1+i)^n}, \tag{3.19}$$

where, C is the coupon payment, i is the interest rate or yield, n is the number of payments， M is the value at maturity and $C+M$ is equal to the nominal value.

In order to understand the description above, we make an example of BBB bond. Assuming that BBB bond has a five-year maturity, and pays annual coupons at the rate of 3%. The face value of this bond is $100. Therefore, the bond pays $3 each at the end of the next three years. *Tab 3.4* shows the one-year forward zero curves by credit rating category (%).

Tab 3.4 One-year forward zero curves by credit rating category (%)

Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 27p.

Assuming the BBB bond upgrades to single-A, we use (3.19) to calculate the value of bond at the end of one year:

$$V = 3 + \frac{3}{(1+3.72\%)} + \frac{3}{(1+4.32\%)^2} + \frac{3}{(1+4.93\%)^3} + \frac{103}{(1+5.32\%)^4} = 94.96,$$

Then we can use the same formula to calculate other value of the bond in a rating category, the results can be obtained in *Tab 3.5*.

Tab 3.5 Possible one-year forward values for a BBB bond plus coupon

Year-end rating	Value($)
AAA	95.62
AA	95.46
A	94.96
BBB	93.93
BB	88.77
B	85.10
CCC	71.92
Default	51.13

Step 3: Valuation of discount rate

Discount rate is the interest rate you earn today under a given amount so that you can reach a certain amount in the future. The discount rate is the time value of the currency. Using discount rate can help bank to make decision to do business with each kind of firms. There should be a line (the deriving risky yield curve) to be the transition probabilities of defaulting rate. There are two steps to obtain this line. First step is from one-year transition matrix to infer the n-year transition probabilities; second step is to use n-year transition probabilities to derive the adjusted rates.

First step, we assume the all probabilities will be zero, the firm is default and the last column is 1.

Firstly, we collect all elements in the matrix:

$$T = \begin{bmatrix} T_v & t_d \\ 0' & 1 \end{bmatrix}, \tag{3.20}$$

where, T_v represents the transition matrix, t_d is the vector of default probability.

Then we consider to calculate two-year transition matrix, and it can be $T \cdot T$.

$$T^2 = T \cdot T = \begin{bmatrix} T_v^2 & (1 + T_v)t_d \\ 0' & 1 \end{bmatrix}, \tag{3.21}$$

The *n-year* transition matrix can be written as follows：

$$T^n = \begin{bmatrix} T_v^n & \sum_{i=0}^{n-1} t_d T_r^i \\ 0' & 1 \end{bmatrix}, \tag{3.22}$$

note T^n means the *n-year* default probabilities for a firm for all rating classes.

The discount rate can be expressed by risk-free rate and teh implicit expectations theory, the equation can be written as:

$$f_t = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1, \tag{3.23}$$

In the second step, we use the *n-year* default probabilities to compute risk-adjusted yield curves.

$$(1+ r_1^i)(1\text{-}p_1^i)\text{+}p_1^i R = 1\text{+}r_1^F, \tag{3.24}$$

where, r_1^i denotes the one-year interest rate, R is the expected recovery rate, r_1^F is the one-year risk-free rate and p_1^i represents probability of default within the first year for i rating class.

It is necessary to calculate the two-year interest rate (r_2^i), the formula can be computed as follows:

$$p_1^i R\frac{(1+r_2^F)^2}{(1+r_1^F)} + (p_2^i - p_1^i)R + (1 + r_2^i)^2\left(1 - p_2^i\right) = (1 + r_2^F)^2 , \tag{3.25}$$

$$r_2^i = \sqrt{\frac{(1+r_2^F)^2 - p_1^i R\frac{(1+r_2^F)^2}{\left(1+r_1^F\right)} - (p_2^i - p_1^i)R}{1-p_2^i}} - 1, \tag{3.26}$$

note r_2^F denotes the two-year risk free rate, p_2^i and p_1^i are the probability of default within the second and first year.

The last equation calculates the generic risk-adjusted rate for borrowers with i class and maturity n (r_n^i). The equation is shown as follows:

$$r_n^i = (1\text{+}r_n^F)\left\{\frac{1-R\sum_{j=1}^{n}\frac{p_j^i - p_{j-1}^i}{(1+r_j^F)^j}}{1-p_n^i}\right\}^{1/n} - 1, \tag{3.27}$$

where r_n^F is the *n-year* risk free rate, p_j^i and p_{j-1}^i represent the probability of default in the j and *j-1* year.

Step 4: Credit risk estimation

In previous two steps, we calculate the results that we need. Then in this step, we need to estimate the volatility of value of the change of credit quality for a single exposure. We get the likelihood and the distribution of value of each outcome, the results are shown in *Tab 3.6*.

Tab 3.6 Calculating volatility in value due to credit quality changes

Year-end rating	Probability of state (%)	New bond value plus coupon ($)	Probability weighted value ($)	Difference of value from mean($)	Probability weighted difference squared
AAA	0.02	95.62	0.02	2.20	0.0010
AA	0.33	95.46	0.32	2.03	0.0136
A	5.95	94.96	5.65	1.54	0.1405
BBB	86.93	93.93	81.65	0.51	0.2223
BB	5.3	88.77	4.70	-4.66	1.1494
B	1.17	85.10	1.00	-8.33	0.8113
CCC	0.12	71.92	0.09	-21.50	0.5548
Default	0.18	51.13	0.09	-42.29	3.2195
		mean =	93.42	Variance =	6.112
				Standard deviation=	2.47

There are two useful measures of credit risk: standard deviation and percentile level. First we consider the standard deviation. To obtain the standard deviation, we should get the average value (the mean). Then we can calculate the dispersion between the individual values and this mean. After that calculation, we can get the standard deviation of the value changes. There are two equations to calculate the mean value (μ) and standard deviation (σ):

$$\mu_{Total} = \sum_{i=1}^{S} p_i \mu_i \,, \tag{3.28}$$

$$\sigma_{Total} = \sqrt{\sum_{i=1}^{S} p_i \mu_i^2 - \mu_{Total}^2} \,, \tag{3.29}$$

where, p_i is the probability of being in any given state, μ_i is the value within each state.

From the *Tab 3.6*, we can obtain mean value (μ_{Total} = 93.42) and standard deviation (σ_{Total} = 2.47) by using the (3.28) and (3.29).

The second measure is the calculation of percentile level. The interpretation of the percentile level is the lowest value of portfolio will achieve 1% of the time the first hundred percentile. The selected confidence level can be as : 95% or 99%. We can do the table to rewrite the probability weighed value in an ascending order, *Tab 3.7* shows the new arranged valuation and cumulative probability.

Tab 3.7 Valuation and cumulative probability

Year-end rating	Difference of value from mean($)	Probability of state (%)	Cumulative probability (%)	New bond value plus coupon ($)
Default	-42.29	0.18	0.18	51.13
CCC	-21.50	0.12	0.30	71.92
B	-8.33	1.17	1.47	85.10
BB	-4.66	5.30	6.77	88.77
BBB	0.51	86.93	93.7	93.93
A	1.54	5.95	99.65	94.96
AA	2.03	0.33	99.98	95.46
AAA	2.20	0.02	100	95.62

In *Tab 3.7*, we can see the likelihood of defaulted state is 0.18%. The probability is less than 1%, then we move up to CCC state. The combined likelihood of CCC state is 0.3% and it is also below 1%, so we move up to B state. The combined likelihood of B state is 1.47%. It exceeds 1%. Therefore, we stop at B state. The value of B state is 85.10 and it is the 1^{st} percentile level value. This is 8.33 below the mean value.

The previous parts introduce the single exposure to the multiple exposure. For more deeply understanding, we can extend to a portfolio of two exposures. Assuming there are two specific bonds. One is a BBB-rated bond which is the senior unsecured and has 3% annual coupon with five-year maturity. Another is a A-rated bond which is the senior unsecured, 5% annual coupon with three-year maturity. In this part, we should estimate the joint likelihoods which the contribution to risk brought by the effects of non-zero credit quality correlations. *Tab 3.8* shows the joint migration likelihood of two obligors with BBB-rated bond and A rated bond with 0.30 asset correlation.

Tab 3.8 Joint likelihood with 0.30 asset correlation (%)

Obligor #1 (BBB)		**Obligor #2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	Default
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.12	0.00	0.00	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 38p.

Then we calculate the A-rated bond plus coupon by using the same way with the BBB-rated bond plus coupon like *Tab 3.5*. The *Tab 3.9* shows the all the likelihood 64 year-end value of two obligors.

Tab 3.9 All the likelihood of 64 year-end value of two bonds

Obligor 1 (BBB)		Obligor 2 (single-A)							
		AAA	AA	A	BBB	BB	B	CCC	D
		103.70	103.61	103.42	102.77	100.31	98.58	86.09	51.13
AAA	95.62	199.33	199.23	199.05	198.39	195.94	194.20	181.71	146.75
AA	95.46	199.16	199.06	198.88	198.23	195.77	194.03	181.54	146.59
A	94.96	198.66	198.57	198.38	197.73	195.27	193.53	181.05	146.09
BBB	93.93	197.63	197.54	197.35	196.70	194.24	192.50	180.02	145.06
BB	88.77	192.47	192.37	192.19	191.53	189.08	187.34	174.85	139.90
B	85.10	188.80	188.70	188.52	187.86	185.41	183.67	171.18	136.23
CCC	71.92	175.62	175.53	175.34	174.69	172.24	170.50	158.01	123.05
D	51.13	154.83	154.74	154.55	153.90	151.44	149.71	137.22	102.3

Now we focus on the calculation of the two risk measures for the portfolio. First we calculate the standard deviation. We can use (3.28) and (3.29) to obtain the mean μ and standard deviation σ. It can be calculated as follow:

$$\mu = \sum_{i=1}^{S=64} p_i \mu_i = 196.7,$$

$$\sigma = \sqrt{\sum_{i=1}^{S=64} p_i \mu_i^2 - \mu^2} = 2.79.$$

The second measures of credit risk is the percentile level. The likelihood of all value should be less than 1%. From the *Tab 3.8* and *Tab 3.9*, it is simple to obtain a 1^{st} percentile level number of \$188.52. This is \$8.25 below the mean value.

Marginal risk

We know how the credit risk can be calculated the credit risk of stand-alone exposure and credit risk for two exposure. Here, the more relevant calculation, marginal risk as the contribution of an asset to total portfolio risk, while others are defined as the marginal effect of marginal effects on portfolio risk. Marginal risk can be using for the standard deviation as a risk measure and can calculate the percentile levels.

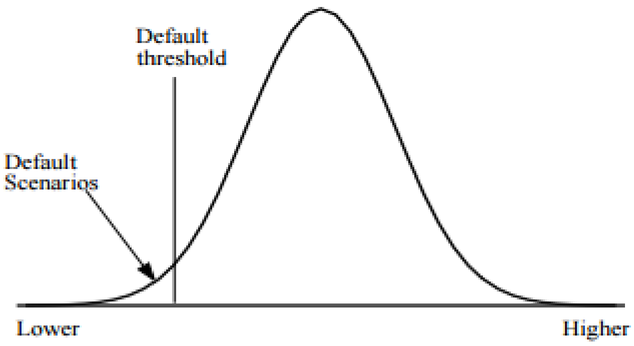
First, we analyze the standard deviation as the credit risk measure. The standard deviation of BBB-rated bond is \$2.47, while the portfolio increases to \$2.79. The marginal standard deviation of the added the single-A rated bond is \$0.32, which is the difference between \$2.47 and \$2.79. The marginal standard deviation is smaller than the single-A rated bond because the year-end values of the individual bonds are not perfectly correlated.

When using percentile levels, the mean value of BBB-rated bond is \$93.42 and a 1^{st} percentile level value is \$85.10. The percentile level is \$8.33 below then mean. The mean of second two-bond portfolio is \$196.77 and a 1^{st} percentile level value is \$188.52. The percentile level is \$8.25 below the mean. The marginal risk of the single-A rated bond is \$0.08, which represents the difference between \$8.33 and \$8.25.

3.2.2 Credit quantity correlation

In this subchapter, credit quantity correlation will be described. Previous part, we use a zero correlation like *Tab 3.8*. It is not realistic to assume a zero correlation because the company's rating changes and defaults are the result of common factors such as economic cycles, changes in interest rates, changes in commodity prices, etc. We can see in *Figure 3.7* is a default as a function of the potential (and unstable) value of the firm. It describes If the value of the asset falls, the value is less than the amount of the outstanding debt, we call it the default threshold, then the company will not be able to fulfill its obligations and will default.

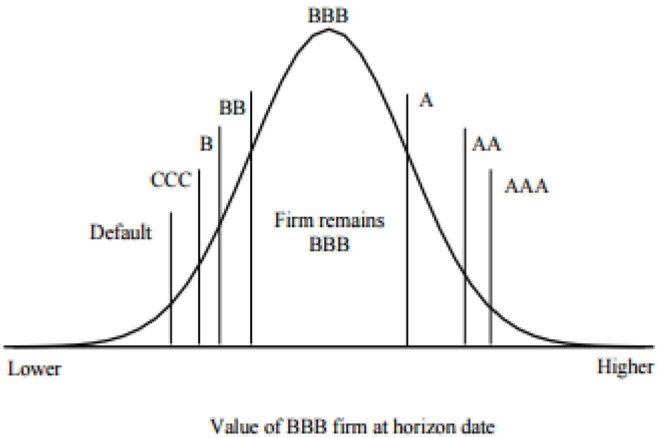
Figure 3.7 Model of firm value and its default threshold



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 37p.

The model can easily extend, for instance, rating changes. It includes the default threshold and the credit rating up(down)grade thresholds. *Figure 3.8* shows the model of firm value and migration.

Figure 3.8 Model of firm value and migration



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 67p.

Now, there arte some equations of partial calculation will be introduced.

First we need calculate the discrete return of shares. The formula can be written as :

$$R_i = \frac{P_t - P_{t-1}}{p_{t-1}}, \tag{3.30}$$

where, R_i = return of the asset, p_{t-1} = value of asset at time *t-1*, P_t = value of asset at time *t*, *t* = time.

The excepted asset return is given by the formal:

$$E(R_i) = \frac{1}{T}\sum_{t=1}^{T} R_i, \qquad (3.31)$$

note $E(R_i)$ is the expected return, T is the number of observations.

Then we calculate the weighted average of expected value of each asset, the equation is shown as follow:

$$E(W) = \sum_i^N w_i \cdot E(R) = \vec{w}^T \cdot E(\vec{R}), \qquad (3.32)$$

where, w_i is the amount of money invested into *i-th* asset, *N* is the number of asset of the portfolio, $\vec{w}^T$ and $E(\vec{R})$ are the vectors.

In order to obtain the variance of overall portfolio, we need know the population variance of returns, the equation can be expressed as:

$$\sigma_i^2 = \frac{1}{N} \cdot \sum_{t=1}^{N} [R_{i,t} - E(R_i)]^2, \qquad (3.33)$$

The variance of overall portfolio can be shown as:

$$\sigma_P^2 = \sum_i^N \sum_j^N w_i \cdot \sigma_{ij} \cdot w_j = \vec{w}^T \cdot C \cdot \vec{w}, \qquad (3.34)$$

where w_i is the weight of *i-th* asset, σ_{ij} is the covariance between assets *i* and *j*, w_j is the weight of *j-th* assets, $\vec{w}$ is the vectors and *C* is the covariance matrix.

The population correlation between two assets ($R_i$and R_j) can be given as:

$$\sigma_{ij} = \frac{1}{N} \cdot \sum_{t=1}^{N} [R_{i,t} - E(R_i)] \cdot [R_{j,t} - E(R_j)], \qquad (3.35)$$

Correlation between two assets is shown as:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j}, \qquad (3.36)$$

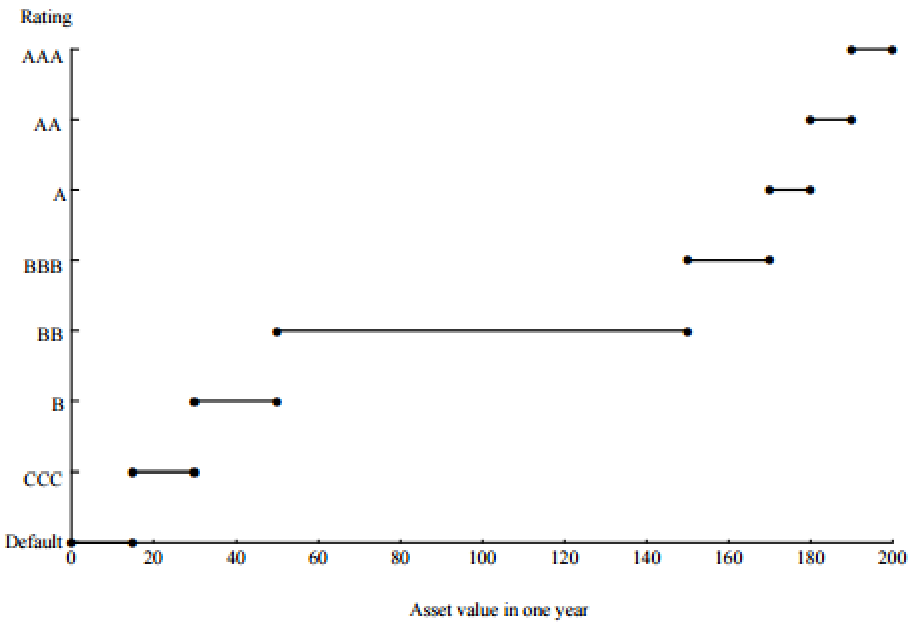
For the single issuers, correlation can be determined by the matrix. We can establish the correlation matrix C (i+j , i+j), and it can be expressed as follow:

$$C = \begin{bmatrix} \sigma^2(Y_1) & \sigma^2(Y_1, Y_2) & \cdots & \sigma^2(Y_1, Y_j) \\ \sigma^2(Y_2, Y_1) & \sigma^2(Y_2, Y_2) & \cdots & \sigma^2(Y_2, Y_j) \\ \vdots & \vdots & \vdots & \vdots \\ \sigma^2(Y_i, Y_1) & \sigma^2(Y_i, Y_2) & \cdots & \sigma^2(Y_j) \end{bmatrix}. \tag{3.37}$$

Asset value model

In order to determine the joint likelihoods (such as upgrades, downgrades and default) of credit quality migrations, we can use asset value model. This model describes the changes of asset values to rating changes because the value of company's asset can determine the ability to pay back its holder. If the company's asset is below the specific level, it will not able to meet its obligation and will default. To understand deeply, we assume a company is BB-rated and its assets are worth \$100 million now. Then we can establish a chart from asset value in a year to rating in a year. *Figure 3.9* shows the credit rating migration by BB-rated company asset value.

Figure 3.9 Credit rating migration by BB-rated company asset value



Source: Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 86p.

From this chart, higher asset value of a company, the better of the credit rating.

Now we assume we know the asset threshold of a company, we can describe the evolution of its credit rating by modeling the changes of company's asset value. There are some parameters should be determined, percent changes in asset value are normally distributed which will denote by R, the mean denoted by μ, and the standard deviation denote by σ. We assume μ = 0 because the value of μ does not influence the final results. Continuing to using BB-rated obligor, there is a transition matrix (*Tab 3.10*) of the obligor form one year. It includes transition probabilities and probability from the asset value model. Using Z_{Def}, Z_{BB}, Z_A, etc., as the asset return thresholds. If R < Z_{Def}, the company will go to default; if Z_{CCC} < R < Z_B, it means the company is downgrade to B, and so on.

Tab 3.10 One year transition probabilities for BB-rated obligor

Rating	Probability from the transition matrix (%)(	Probability according to the asset value model
AAA	0.03	$1-\Phi(Z_{AA}/\sigma)$
AA	0.14	$\Phi(Z_{AA}/\sigma)-\Phi(Z_A/\sigma)$
A	0.67	$\Phi(Z_A/\sigma)-\Phi(Z_{BBB}/\sigma)$
BBB	7.73	$\Phi(Z_{BBB}/\sigma)-\Phi(Z_{BB}/\sigma)$
BB	80.53	$\Phi(Z_{BB}/\sigma)-\Phi(Z_B/\sigma)$
B	8.84	$\Phi(Z_B/\sigma)-\Phi(Z_{XXX}/\sigma)$
CCC	1.00	$\Phi(Z_{XXX}/\sigma)-\Phi(Z_{Def}/\sigma)$
Default	1.06	$\Phi(Z_{Def}/\sigma)$

Souce: Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 87p.

There are some formulas to calculate the probability of each event:

$$Pr\{Default\} = Pr\{R < Z_{Def}\} = \phi(Z_{Def} / \sigma), \tag{3.38}$$

$$Pr\{CCC\} = Pr\{Z_{Def} < R < Z_{CCC}\} = \phi(Z_{CCC} / \sigma) - \phi(Z_{Def} / \sigma), \tag{3.39}$$

note ϕ is the cumulative distribution for the standard normal distribution.

For instance, we consider the CCC-rated probability by using (3.39):

$$Pr\{CCC\} = Pr\{Z_{Def} < R < Z_{CCC}\} = \phi(Z_{CCC} / \sigma) - \phi(Z_{Def} / \sigma) = 1.00\%,$$

The equation of threshold value can be written as:

$$Z_{rating} = \phi^{-1} (p) \cdot \sigma , \tag{3.40}$$

Then we use (3.40). we can get the Z_{CCC}:

$$Z_{CCC} = \phi^{-1} (1.00\%) \cdot \sigma = -2.04\sigma,$$

where $\phi^{-1}(p)$ is the level below which a standard normal distributed random variable falls with probability p. Other rating threshold calculation can use the similar way to do.

We can get the threshold values of asset value for BBB rated obligor in *Tab 3.11*.

Tab 3.11 Threshold values

Threshold	Value
Z_{Def}	-2.30c
Z_{CCC}	-2.04σ
Z_{B}	-1.23σ
Z_{BB}	1.37σ
Z_{BBB}	2.39σ
Z_{A}	2.93σ
Z_{AA}	3.43σ

We can obtain the transition probabilities and asset return of A-rated obligor by using the similar steps with BB-rated obligor. *Tab 3.12* shows the transition probabilities for A-rated obligor.

Tab 3.12 Transition probabilities for A-rated obligor

Rating	Probabilities
AAA	0.09%
AA	2.27%
A	91.05%
BBB	5.52%
BB	0.74%
B	0.26%
CCC	0.01%
Default	0.06%

Using (3.40) and we can get the threshold value of A-rated obligor in *Tab 3.13*.

Tab 3.13 Threshold value of A -rated obligor

Threshold	Value
Z'_{Def}	-3.24σ'
Z'_{CCC}	-3.19σ'
Z'_{B}	-2.72σ'
Z'_{BB}	-2.30σ'
Z'_{BBB}	-1.51σ'
Z'_{A}	1.98σ'
Z'_{AA}	3.12σ'

The formula of the two ratings jointly and the assumption is the two asset returns are correlated and bivariate normally distributed, and there is only the correlation ρ between the two asset returns. The covariance matrix for the bivariate normal distribution can be expressed as:

$$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma\sigma' \\ \rho\sigma\sigma' & \sigma'^2 \end{pmatrix}. \tag{3.41}$$

In order to compute the probability of two obligors based on their current credit rating, we assume the correlation $\rho \neq 0$. The asset return of the BB-rated firm falls between Z_B and Z_{BB} and the asset return of A-rated firm also falls between Z'_{BBB} and Z'_A. The formula of probability of two obligors are shown as follow:

$$Pr\{Z_B < R < Z_{BB}, Z'_{BBB} < R < Z'_A\} = \int_{Z_B}^{Z_{BB}} \int_{Z'_{BBB}}^{Z'_A} f(r,\ r';\Sigma)(dr')dr, \tag{3.42}$$

note $f(r, r'; \Sigma)$ represents the density function for the bivariate normal distribution with covariance matrix, r and r' are the values of two asset returns may within the specified intervals.

Monte Carlo simulations

Monte Carlo simulations are based on the generation of random data. These data come from the historical sample of estimation of the parameters of a particular probability distribution (such as a normal distribution, etc) and allocating N simulated values from the probability extraction risk factor. Cholesky decomposition is a matrix if all the elements above of below the main diagonal are equal to zero. The function of covariance matrix can be decomposed as:

$$\Sigma = \begin{bmatrix} \sigma_A^2 & \sigma_{A,B}^2 \\ \sigma_B^2 & \sigma_B^2 \end{bmatrix} = \begin{bmatrix} \sigma_A & 0 \\ \frac{\sigma_{A,B}^2}{\sigma_A} & \sqrt{\sigma_B^2 - (\frac{\sigma_{A,B}^2}{\sigma_A})^2} \end{bmatrix} \begin{bmatrix} \sigma_A & \frac{\sigma_{A,B}^2}{\sigma_A} \\ 0 & \sqrt{\sigma_B^2 - (\frac{\sigma_{A,B}^2}{\sigma_A})^2} \end{bmatrix} = AA', \quad (3.43)$$

where, σ_A^2 is the variance of A variable, σ_B^2 is the variance of B variable, and $\sigma_{A,B}^2$ is the variance of A,B.

The correlation matrix is computed as follows:

$$\Sigma = \begin{vmatrix} 1 & \rho \\ \rho & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ \rho & (1-\rho^2)^{1/2} \end{vmatrix} \begin{vmatrix} 1 & \rho \\ 0 & (1-\rho^2)^{1/2} \end{vmatrix}. \quad (3.44)$$

To calculate the individual event of the Cholesk decomposition matrix, the equations can be expressed as follows:

$$P_{ii} = (\sigma_{ii} - \sum_{k=1}^{i-1} P_{ki}^2)^{1/2}, \quad i = 1,2,3,\ldots,N, \quad (3.45)$$

$$P_{ij} = (\sigma_{ii} - \sum_{k=1}^{i-1} P_{ki} P_{kj}) \cdot P_{ii}^{-1}, \quad i, j = 1,2,3,\ldots N, \quad (3.46)$$

$$P_{ij} = 0, \quad i > j, \qquad i, j = 1,2,3,\ldots,N, \quad (3.47)$$

note P_{ii} and P_{ij} are the individual elements of Cholesk decomposition matrix

The simulations include three steps to calculate:

Step 1: Generate m random values (such as $p_1, p_2, \ldots p_m$) from 0 to 1;

Step 2: Translate the random values into as many values (such as v_1, v_2, v_3) from a standard normal;

Step 3: Adjust them by using formula $x = v \cdot \sigma + \mu$, and generate x_1, x_2,…, x_m to reflect their true mean and variance.

Monte Carlo simulation has some advantages: Monte Carlo simulation can simulate market factors to evolve and recalculate market value including the position of the entire portfolio; it is more efficiency than other numeric procedures; it can be used for any probability distribution.

3.2.3 Applications

In the previous parts, we do the calculation of one and two-asset portfolios, analyze the value change of the mean and standard deviation of portfolio, and make the discussion of the input. In this subchapter, we will discuss the results we calculate by using CreditMetrics™. The popular way to present the results is using statistical terms, for example standard deviation or percentiles. There are some ways to present the results: a graphic, figure and so on. First, for large portfolio (usually more than two assets), we should calculate the standard deviation and marginal standard deviation. Then considering the distribution of the portfolio value changes. The last is to utilize the risk-taking capital more efficiently. There is a risk line that in order to optimize the return on the risks we receive and it is necessary to help us to identify the risk. There are two ways to reduce the risk of the portfolio: Prioritizing risk reduction actions and credit risk limits.

Prioritizing risk reduction actions

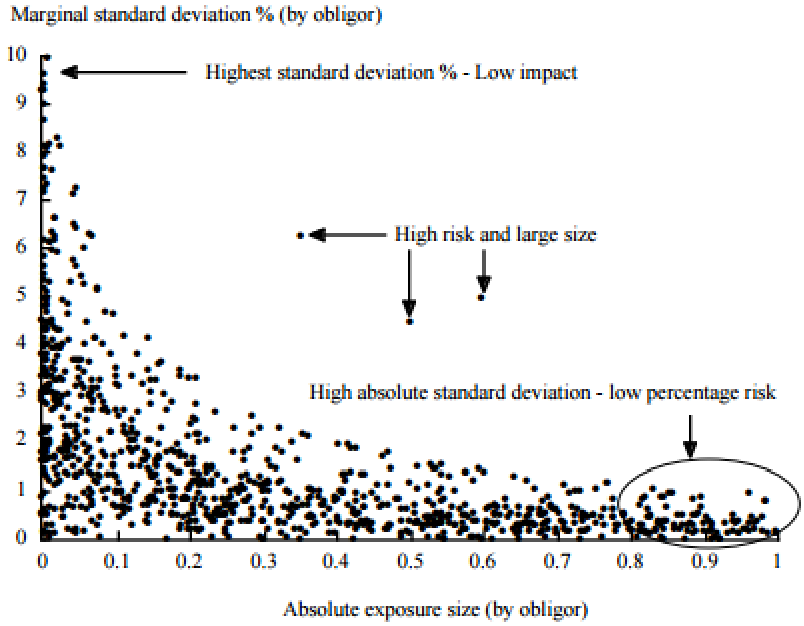
Many actions can be taken to deal with the towards risks and therefore must be prioritized, especially for an assuming portfolio with a very large number of exposures (*Figure 3.10*). Generally, there are two characteristics of risk which are worth reducing : absolute exposure size and statistical risk level. The approaches are:

· Obligors have the largest absolute size (the lower right corner of the *Figure 3.10)* would have the greatest impact than others;

· Obligors have the highest percentage level of risk (the upper left corner of the *Figure 3.10*) shows the most likely to contribute to the portfolio losses;

· Obligors have the largest absolute amount of risk (points towards the upper right corner of the *Figure 3.10*) shows the single largest contributors to portfolio risk.

Figure 3.10 Risk versus size of exposures within a typical credit portfolio



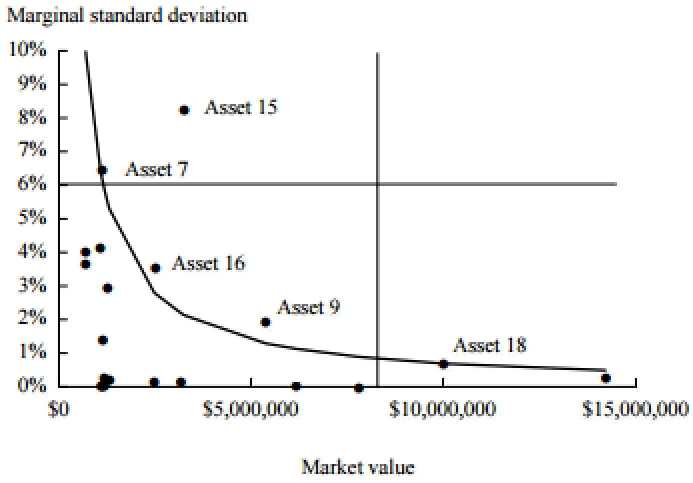
Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 134p.

The third one is the most using to set the highest priority as a debtor with a higher percentage of risk and a relatively high risk. If the obligors have the high percentage risk, it can be tolerated when there are small. Large exposures are only allowed if they have relatively small percentage risk levels. However, when the exposures are created large and they have higher percentage to credit rating downgrades.

Credit risk limits

This step is to make the credit risk limits. There are three limit purpose for CreditMetrics: what type of credit risk limits, choice of risk measure, and the policy issues.

Figure 3.11 Possible risk limits for an example portfolio



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 135p.

Figure 3.11 above shows the possible risk limits for an example portfolio. It includes three possibilities for credit risk limits:

· based on percentage risk. It corresponds as a horizontal line (in Figure 3.11). Due to use the marginal term to measure the risk, the limit would restrict exposures are more correlated to the portfolio;

· based on exposure size. It corresponds as a vertical line in Figure 3.11. The limit restricts the portfolio without exposures and credit quality above a given size;

· based on absolute risk. It corresponds as a curve in Figure 3.11. The limit is to prevent the risk of portfolio losses by more than a given amount of any risk of the portfolio. The cap of the total risk of the portfolio would above the current risk at a certain amount.

The second purpose is the choice of risk measure. It uses marginal statistics to examine an exposure of the actual portfolio and takes into accounts the correlation and diversification.

As to use statistic tools, it includes four statistics: standard deviation—the easiest statistics; percentile level—easily to define and it has a very concrete meaning; average shortfall—it defines the expected loss at the time of a given loss; expected excession of a percentile level—it defines the expected loss is more extreme than 1st percentile level.

The third purpose is the policy issues. It can be classified according to the severity of the action taken in the case of exceeding the limit.

3.3 Regulation of capital requirements

When banks develop their own risk management systems to measure the amount of capital occupied by their risky assets, they must also consider restrictions on applicable regulations. The regulatory of the mandatory capital requirement proposed by the Basel Committee in 1988. At first, the capital requirement only focused on the credit risk, along with the development of economy and the system of bank, it extended to market risks in 1996 (Basel II) and operational risks in 2004 (Basel Ⅲ).

3.3.1 Basel I

In December 1987 the regulatory requirements based on capital ratios were proposed by the Basel Committee. Basel Committee is an advisory created by the 10 largest industrialized countries: Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom and the United State, plus Luxembourg and Spain. In July 1988 the Capital Accord ratified, and was implement in 1992.

Based on 1988 Accord, capital has two tiers:

Tier 1 is the core capital of the bank. It includes share premium account, legal reserve, retained earnings, common stocks and surplus, minority interest in the equity accounts of consolidated subsidiaries and intangible goods.

Tier 2 is the supplementary funds, it is not reliable than Tier 1. It includes undisclosed reserves, the allowance for loan and lease loans, revaluation reserves, hybrid capital, and other subordinated term debt.

Now we can consider the level of capital requirement, they can be written as:

$$\textit{Tier 1 ratio} = \frac{Tier\ 1}{RWA} \geq 4\%, \tag{3.48}$$

$$\textit{Capital requirement ratio} = \frac{Tier\ 1 + Tier\ 2}{\sum_i A_i \cdot w_i} \geq 8\%, \tag{3.49}$$

where A_i is *i-th* asset, w_i denotes risk weight of the *i-th* asset.

The regulatory capital of a bank based on 1988 Accord is at least 8% of total risk-weighted assets.

Credit risk exposure can be divided into three parts: those are in on-balance sheet assets, off-balance sheet assets and over-the counter derivatives (only OTC market includes derivatives). Here we focus on the on-balance sheet assets. There are four risk weights classes for on-balance sheet assets:

w_i = 0%: it usually includes cash and cash equivalents, claims on central bank and central government of OECD[2] countries and government bonds issued by OECD countries;

w_i = 20%: it concludes claims on banks in OECD countries such as short-term securities);

w_i = 50%: it includes loans secured by mortgage on residential property;

w_i = 100%: it consists of claims on banks and government outside the OECD[1]

The risk-weighted asset for N items can be expressed as follows;

$$RWA = \sum_{i=1}^{N} w_i \cdot EAD_i, \tag{3.50}$$

note w_i is the risk weight of the *i-th* asset, EAD_i is the exposure at default of *i-th* asset.

Basel I concentrated on credit risk till 1996, market risk was introduced, therefore, the capital adequacy ratio(CAR) can be calculated as follows:

$$Capital\ adequacy\ ratio = \frac{Tier\ 1 + Tier\ 2}{RWA + (CR_m \cdot 12.5)} \geq 8\%, \tag{3.51}$$

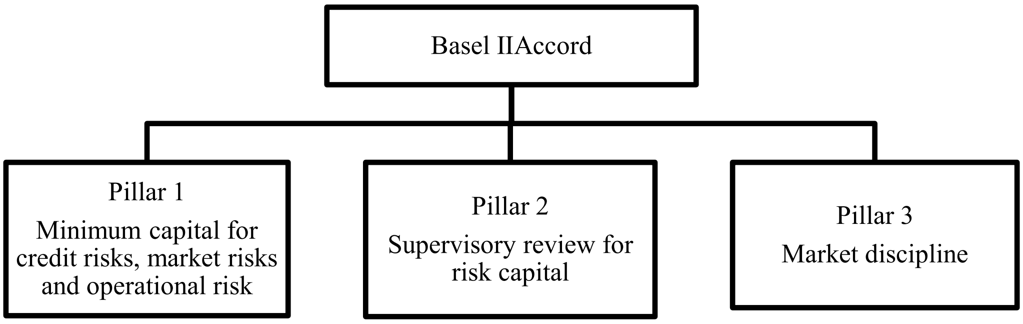
where, CR_m is the capital requirement for market risk.

3.3.2 Basel II

The Basel Committee issued a new capital framework to replace the 1988 Accord. The new accord suggests that capital requirements be more closely linked to the risks actually assumed by banks. It is also intended to expand the risk that banks consider when calculating their minimum capital requirements. The new accord encourages banks to develop more complex internal risk management systems to reduce the non-systemic risk in the banking system. In 2004, a new capital framework Basel IIwas implemented. It includes three pillars (*Figure 3.12*):

[2] OECD is the Organization for Economic Co-operation and Development.

Figure 3.12 The three pillars of the Basel Ⅱ Accord



Pillar 1 Minimum capital requirement

The minimum capital requirement is reflected on three major types of risks in bank, such as credit risks, market risks and operational risk. The new minimum capital requirement of the three major risks is considered. The total risk-weighted assets (RWA_T) for the bank can be rewritten as follows:

$$RWA_T = RWA_C + 12.5 \cdot (CR_m + CR_o), \tag{3.52}$$

where, RWA_C is the total risk-weighted assets for credit risk, CR_m represents the market risk capital requirement, CR_o is the operational risk capital requirement.

The regulatory capital (RC) can be expressed as follows:

$$RC = Tier\ 1\ Capital + Tier\ 2\ Capital - Deductions, \tag{3.53}$$

The minimum capital requirement of Basel II can be stated as:

$$Capital\ requirement\ ratio = \frac{RC}{RWA_T} \geq 8\%, \tag{3.54}$$

Tier 3 capital is the subordinated loans with maturity longer than two year and it usually does not consider in regulatory capital. For credit risk, Tier 1 and Tier 2 capital of credit risk must be higher than 8% of RWA_C; for market risk, Tier 1, Tier 2 and Tier 3 capital must be greater than CR_m.

Basel II encourages the banks to develop more complex internal risk management systems to estimate the banks' riskiness. There are three methods of calculating capital according to Basel II (*Tab 3.14*):

Tab 3.14 Methods for calculating capital according to Basle II

	Credit Risk	Market Risk	Operational Risk
Approaches	Standardized Approach	Standardized Approach	Basic Indicator Approach
	Foundation internal Ratings-Based (IRB) Approach	Internal Models Approach	Standardized Approach
	Advanced IRB Approach		Advanced Measurement Approach
Result	Risk-weighted asset value for credit risk	Market risk capital charge	Operational risk capital charge

Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 203p.

Standardized Approach

In Basel II proposes that the risk of certain asset types is weighted, especially for sovereign countries (countries) and banks, and that the risk rating is determined by the external credit rating assigned to the borrower. In Standardized Approach, different categories of counterparts (such as non-financial companies, countries, banks) to give different weight ratio. *Tab 3.15* shows the Risk weights in the standard approach.

Tab 3.15 Risk weights in the standard approach

	Corporates	Public Sectors	Banks	Government
Unrated	100%	100%	100%	100%
under B-	150%	150%	150%	100%
B+ to B-	150%	150%	150%	100%
BB+ to B+	100%	100%	100%	100%
BBB+ to BBB-	100%	100%	100%	50%
A+ to A-	50%	50%	50%	20%
AAA to AA-	20%	20%	20%	0%

From the *Tab 3.15*, we can know the risk weights of government is from 0% to 100%, the risk weights of other industries (corporates, public sectors and banks) is from 20% to 150%. For an example to understand clearly, if a loan of \$200 to a corporate with a AAA to AA- rating, it means a corporate has \$20 risk-weighted assets. Then a capital requirement is \$20 · 8% = \$1.6, it means there is 1.6% of the non-weighted exposure.

Internal Rating-based Approach (IRB)

Basel II refers other two approaches to estimate minimum credit capital requirement. Banks can use their internal systems to generate the credit ratings. The two approaches are: Foundation Internal Rating-based (IRB) Approach and Advanced Internal Rating-based Approach.

In IRB approach, banks rely on their own information to determine the minimum capital requirements. The bank evaluates the borrower's internal processes to generate information to create a credit model. To build the credit model, there are four risk factors should be considered, that probability of default (PD), exposure at default (EAD), loss given default (LGD) and maturity (M).

For foundation approach, LGD is always 45% for all unsubordinated and unsecured exposures, and the LGD of the subordinated loans of banks can be 75%, the maturity is 2.5 years. For advanced approach, banks can use their internal system to calculate the EAD and LGD. The formulas of internal rating-based approach for risk-weighted assets (if the exposures are not in default) can be written as follows:

$$RWA = CR \cdot 12.5 \cdot EAD, \tag{3.55}$$

where, CR is the capital requirement.

$$Capital\ requirement\ (CR) = \left[LGD \cdot N\left(\sqrt{\frac{1}{1-R}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right) - PD \cdot LGD\right] \cdot \frac{1+(M-2.5)\cdot b}{1-1.5\cdot b}, \quad (3.56)$$

note *N(x)* denotes the nominal distribution for a standard normal variable with N(0,1), *G(z)*is the inverse function for a standard nominal variable such that *N(x) = z*, *0.999* is the confidence level, $PD \cdot LGD$ represents the expected loss, $1.5 \cdot b$ is the special factor to maturity.

The correlation (R) for retail banking can be expressed as follows:

$$R = 0.12\cdot \frac{1-EXP(-50\cdot PD)}{1-EXP(-50)} + 0.24 \cdot \left[1 - \frac{1-EXP(-50\cdot PD)}{1-EXP(-50)}\right], \quad (3.57)$$

$$Maturity\ adjustment\ \ (b) = [0.11852 - 0.05478 \cdot ln(PD)^2], \quad (3.58)$$

The correlation (R') for commercial banking can be calculated as follows:

$$R' = 0.12\cdot \frac{1-EXP(-50\cdot PD)}{1-EXP(-50)} + 0.24 \cdot \left[1 - \frac{1-EXP(-50\cdot PD)}{1-EXP(-50)}\right] \cdot 0.04\cdot \left(1 - \frac{S-5}{45}\right), \quad (3.59)$$

where *S* is the sales.

Pillar 2: Supervisory Review

Pillar 2 lists the principles of the supervisory review process for assessing the bank's capital adequacy ratio by the national authorities. It involves the interaction between national regulators and banks in order to take into account the specific risk characteristics of individual banking institutions. Regulators should supervise the banks to hold the more capital requirement in a balance level to avoid the risk. Moreover, the regulators also focus on the organizational layout of banks, the management quality of banks and so on.

Pillar 3: Market Discipline

The market discipline guides investors and markets and gives bank customers a clearer understanding of bank operations and exposure. A very important event for the investors and markets of the corporates is the financial statements. Pillar 3 concentrates the capital information of a corporate (also banks): capital structure; capital adequacy and risk exposure. Under the Pillar 3, banks should make their disclosures for every six months.

3.3.3 Basel III

From 2010 to 2011, members of the Basel Committee on Banking Supervision agreed on the accord and planned to implement between 2013 and 2015. But on April 1, 2013, the implementation of the accord extended to March 31, 2018, and once again extended to March 31, 2019. Basel Accord "is formulated for the financial regulatory flaws revealed by the 2007-2008 financial crisis. Basel III aims to strengthen bank capital requirements by increasing bank liquidity and reducing bank leverage. *Tab 3.16* shows some changes between Basel IIand Basel III.

Tab3.16 The changes between Basel IIand Basel III

Basel II	Requirements	Basel III*
8%	Minimum Ratio of Total Capital to RWAs	10.50%
2%	Minimum Ratio of Common equity to RWAs	4.5% to 7%
4%	Tier 1 Capital to RWAs	6%
2%	Core Tier 1 Capital to RWAs	5%
None	Capital Conservation Buffer to RWAs	2.50%
None	Leverage Ratio	3%
None	Countercyclical Buffer	0% to 2.5%
None	Minimum Liquidity Coverage Ratio	TBD (2015)
None	Minimum Net Stable Funding Ratio	TBD (2018)
None	Systemically Important Financial Institutions Charge	TBD (2011)

* Basel III requirements will be progressively phased-in over the next eight years.

Source : http://riskarticles.com/basel-iii-key-updates/

There are some new provisions in Basel III:

· banks must always maintain a minimum level of Tier 1 capital ratio of 4.5%;

·capital conservation buffer should be equal to 2.5% of risk-weighted assets, the minimum capital adequacy ratio must be increased to 13% until 2019;

· tier 1 leverage ratio must be greater or equal to 3%;

· two new liquidity ratios are introduced: liquidity coverage ratio which Requires banks to hold sufficient high-quality current assets to cover the total net cash outflows within 30 days; net stable funding ratio, which the amount available to stabilize the funds exceeds the required amount of stable funds during the extended period of one year.

4 Determination of credit risk by selected models

In the previous chapter, we have described the financial risk, credit risk and some models. To understand deeply, in this chapter, we will use some models to identify the results—capital requirement, which can be calculated by Basel agreements and Creditmatrics model.

The aim of this thesis is to compute and compare the value of capital requirement for unexpected losses based on credit risk of ten debt assets portfolio under Basle I, Basel II and Basel III. Also, we use CreditMetrics™ model to calculate the economic capital.

First, the basic data and portfolio will be input. Time horizon of our calculation is aimed to cover the unexpected loss is one year and begins on January 1st, 2017. We choose ten different bonds from ten companies. Then using the Basel agreements, standard approach (SA) and foundation internal rating-based approach (FIRB) to calculate the capital requirement. Economic capital will be presented by Creditmatrics model. At the end, we compare the results of two methods.

4.1 Input data

The input data is about ten different debt assets listed on Frankfurt Stock Exchange (FSE) with total nominal value of 10 million euro (we assume). Each bond is represented in 1 million euro to avoid bias caused by high nominal value of some debts. The *Tab 4.1* show the basic information of input data. It includes the rating, maturity, market price, coupon, nominal value and pcs.. All bonds are senior unsecured because these bonds is by the issuer's full trust and credit guarantee. The information of bonds can find in the Frankfurt Stock Exchange.

In *Tab 4.1*, all bonds are denominated in euros (€). The nominal value of bonds are between 1,000€ to 200,000€. The ratings are provided by Standard & Poor's, the highest rating is AAA and lowest rating is BBB-. Market prices of each ten bonds are presented in the table below and the prices are from 102.90% to 117.64%. We also found the coupon rates of each ten bonds and the highest coupon rate is Nokia which the credit rating is BB+ with the maturity is 12/2019 because the rating of Nokia is the lowest in our data and the it need to attract more investors to invest its's company.

Tab 4.1 The basic information of data

Name	**Rating**	**Maturity**	**Market price**	**Coupon**	**Nominal value**	**pcs.**
Toyota Motor	AAA	02/2023	111.29%	2.38%	1,000	1,000
Renault	BBB-	09/2018	105.49%	3.63%	1,000	1,000
Nokia	BB+	02/2019	112.34%	6.75%	50,000	20
Vodafone Group	BBB+	06/2022	117.64%	5.63%	100,000	10
The Royal Bank of Scotland	A-	09/2020	115.60%	5.50%	50,000	20
SoftBank	BB+	07/2025	109.24%	6.00%	200,000	5
Sydney Airport	BBB	04/2024	110.59%	2.75%	100,000	10
Pepsi	A+	04/2021	106.40%	1.75%	100,000	10
Philip Morris	AA	03/2020	105.00%	1.75%	100,000	10
Berlin, Land	AA+	07/2026	102.90%	1.25%	1,000	1,000

Source: Frankfurt Stock Exchange

Also, the probability of default of individual bond with different rating should be known. The probability of European companies can be find from Standard & Poor's are shown in *Tab 4.2*.

Tab 4.2 Probability of default

Rating	**PD**	**Rating**	**PD**
AAA	0.0007%	**BBB**	0.2730%
AA+	0.0022%	**BBB-**	0.2747%
AA	0.0024%	**BB+**	0.7117%
AA-	0.0044%	**BB**	1.2581%
A+	0.0142%	**BB-**	4.1917%
A	0.1075%	**B+**	8.8480%
A-	0.2020%	**B**	24.4180%
BBB+	0.2045%	**B-**	48.6187%
		CCC	

Source: Standard & Poor's

The recovery rate can be found in Cath & Lieberman, due to all bonds are senior unsecured because seior unsecured is not guaranteed by a particular asset but is guaranteed by the issuer's full credit, the recovery rate is 51.13% in Chapter 2, *Tab 2.4*. From this table, we can obtain the loss given default by using (2.2).The loss given default is 48.87%.

4.2 Calculate the credit risk under Basel

In this subchapter, we calculate the capital requirement under Basel agreement. The method is described in subchapter 3.3, each bond of portfolio is represented in nominal value of 1 million euro.

Under Basel Ⅰ

At first, we calculate the capital requirement under Basel I. Using (3.48) can easily get risk-weighted assets. Then calculating the capital requirement when obtains the risk-weighted assets. The results are shown in *Tab 4.3*.

Tab 4.3 Capital requirement under Basel Ⅰ

	Rating	Nominal value	w	RWA	CR
Toyota Motor	AAA	1,000,000 €	100%	1,000,000 €	80,000 €
Renault	BBB-	1,000,000 €	100%	1,000,000 €	80,000 €
Nokia	BB+	1,000,000 €	100%	1,000,000 €	80,000 €
Vodafone Group	BBB+	1,000,000 €	100%	1,000,000 €	80,000 €
The Royal Bank of Scotland	A-	1,000,000 €	20%	200,000 €	16,000 €
SoftBank	BB+	1,000,000 €	20%	200,000 €	16,000 €
Sydney Airport	BBB	1,000,000 €	100%	1,000,000 €	80,000 €
Pepsi	A+	1,000,000 €	100%	1,000,000 €	80,000 €
Philip Morris	AA	1,000,000 €	100%	1,000,000 €	80,000 €
Berlin, Land	AA+	1,000,000 €	100%	1,000,000 €	80,000 €
Total	-	-	-	8,400,000 €	672,000 €

The table is the foundation of Basel I. Th weight of all bonds are almost 100% except The Royal Bank of Scotland and Softbank. The weght of The Royal Bank of Scotland and Softbank is 20% because the sector of these two is banking, others are corporates.The value of risk-weighted asset of all bonds is 8,400,000€ and the capital requirement is 672,000€.

Under Basel II

Capital requirement under Basel II can be calculated in two approaches, which are standard approach (SA) and foundation internal rating-based approach (FIRB). The producers of standard approach of Basel II are similar as Basel I and the results are presented in Tab 4.4.

Tab 4.4 Capital requirement under Basel II—SA

	Rating	Nominal value	w	RWA	CR
Toyota Motor	AAA	1,000,000 €	20%	200,000 €	16,000 €
Renault	BBB-	1,000,000 €	100%	1,000,000 €	80,000 €
Nokia	BB+	1,000,000 €	100%	1,000,000 €	80,000 €
Vodafone Group	BBB+	1,000,000 €	100%	1,000,000 €	80,000 €
The Royal Bank of Scotland	A-	1,000,000 €	50%	500,000 €	40,000 €
SoftBank	BB+	1,000,000 €	100%	1,000,000 €	80,000 €
Sydney Airport	BBB	1,000,000 €	100%	1,000,000 €	80,000 €
Pepsi	A+	1,000,000 €	50%	500,000 €	40,000 €
Philip Morris	AA	1,000,000 €	20%	200,000 €	16,000 €
Berlin, Land	AA+	1,000,000 €	20%	200,000 €	16,000 €
Total	-	-	-	6,600,000 €	528,000 €

The Basel II—SA improves the shortage of Basel I.We can see in Tab 4.4, the value of each individual bond changes a lot because of the different risk weights. The value of risk-weighted asset is 6,600,000€ and the capital requirement is 528,000€. The capital requirement of standard approach under Basel II decreases 21.43% compared with Basel I. We can see the banking sector, for example, weight of the Royal Bank of Scotland in Basel I is 20% and the weight in Basel II is 50% because of the different criterias in different Basel aggrement. The absolute change of risk weighted asset between Basel I and Basel II is 300,000€. The relative change of capital requirement is 60%, it means under Basel II, the capital requirement of the Royal Bank of Scotland increases. In the corporate, for example, the weight of Pepsi in Basel I is 100% and the weight in Basel II is 50% based on the different degree of credit rating under Basel II. The absolute change of risk weighted asset between Basel Iand Basel II is -500,000€ because the weight of Pepsi changes. The relative change of capital requirement is -50% and it means the capital requirement under Basel II decreases based on the different weight in different Basel agreements.

Then we use foundation internal rating-based approach to calculate the size of capital requirement. The formulas we can use (3.54), (3.55) and (3.56), it can simply to obtain the value of risk-weighted assets and value of capital requirement under Basel II—FIRB. The results are shown in *Tab 4.5*.

Tab 4.5 Capital requirement under Basel II—FIRB

	Rating	RWA	CR
Toyota Motor	AAA	31,080 €	2,486 €
Renault	BBB-	564,403 €	45,152 €
Nokia	BB+	880,182 €	70,415 €
Vodafone Group	BBB+	482,507 €	38,601 €
The Royal Bank of Scotland	A-	479,292 €	38,343 €
SoftBank	BB+	880,182 €	70,415 €
Sydney Airport	BBB	562,588 €	45,007 €
Pepsi	A+	100,321 €	8,026 €
Philip Morris	AA	39,317 €	3,145 €
Berlin, Land	AA+	37,943 €	3,035 €
Total	-	4,057,814 €	324,625 €

Foundation internal rating-based approach allows bank to build their own credit models to calculate the minimum credit requirement. We can find in *Tab 4.5*, the risk-weighted assets of all bonds decrease obviously and the result is about 4 million. The capital requirement also declines about 20 thousand. Compared with Basel II—SA, for example, Renault, the risk weighted asset is 1,000,000€ in Basel II—SA and the risk weighted asset in Basel II—FIRB is 564,403€. The relative change of risk weighted asset and capital requirement in different approaches under Basel IIare the same as -43.56%. However, foundation internal rating-based approach uses the banks or corporates internal information to determine the capital requirement and this approach is more closed to the actual situation.

Under Basel Ⅲ

Thirdly, we use Basel III agreement to calculate the capital requirement. It also includes standard approach and foundation internal rating-based approach and the procedures are similar as Basel II. However, there is a new provision of capital: the minimum capital adequacy is 13%. In *Tab 4.6*, we can see the capital requirement under Basel III--standard approach. The capital requirement under Basel III—SA increases 31.25% compared with Basel II—SA. The result of risk-weighted asset is the same as Basel II—SA but decreases almost 21% compared with Basel I.

Tab 4.6 Capital requirement under Basel III--SA

	Rating	Nominal value	w	RWA	CR
Toyota Motor	AAA	1,000,000 €	20%	200,000 €	21,000 €
Renault	BBB-	1,000,000 €	100%	1,000,000 €	105,000 €
Nokia	BB+	1,000,000 €	100%	1,000,000 €	105,000 €
Vodafone Group	BBB+	1,000,000 €	100%	1,000,000 €	105,000 €
The Royal Bank of Scotland	A-	1,000,000 €	50%	500,000 €	52,500 €
SoftBank	BB+	1,000,000 €	100%	1,000,000 €	105,000 €
Sydney Airport	BBB	1,000,000 €	100%	1,000,000 €	105,000 €
Pepsi	A+	1,000,000 €	50%	500,000 €	52,500 €
Philip Morris	AA	1,000,000 €	20%	200,000 €	21,000 €
Berlin, Land	AA+	1,000,000 €	20%	200,000 €	21,000 €
Total	-	-	-	6,600,000 €	693,000 €

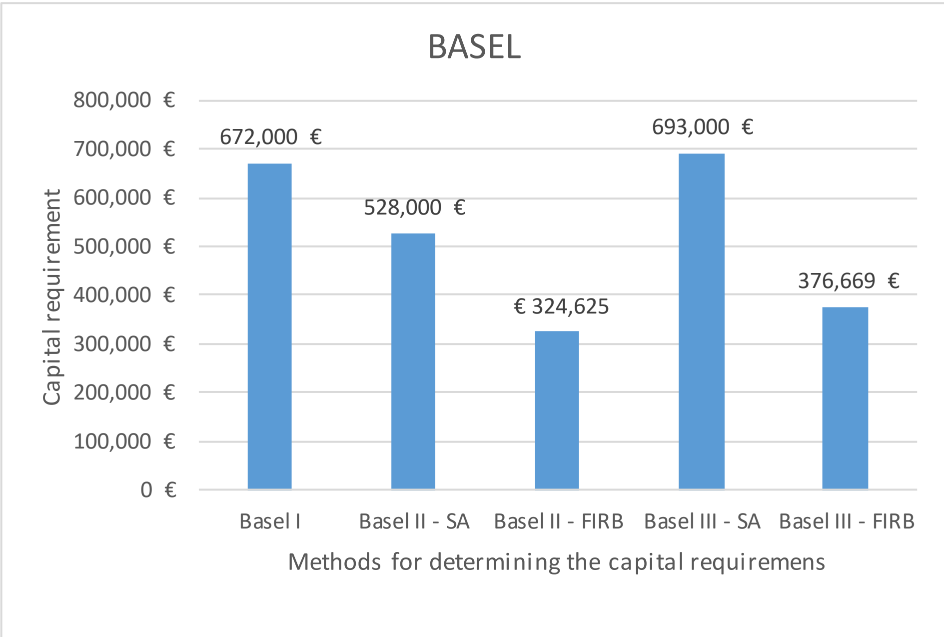
Then we use foundation internal rating-based approach of Basel III to obtain the capital requirement. The producers are similar as the Basel II. The results are presented in *Tab 4.7*. We can see the risk-weighted assets is higher than Basel II—FIRB and the absolute change is 650€. The capital requirement is also higher than Basel II—FIRB and the relative change is about 16%. Comparing with Basel III—SA, we can find the risk-weight assets and capital requirement are higher than the results of Basel III—FIRB.

Tab 4.7 Capital requirement under Basel III--FIRB

	Rating	RWA	CR
Toyota Motor	AAA	1,153 €	92 €
Renault	BBB-	577,984 €	46,239 €
Nokia	BB+	1,321,858 €	105,749 €
Vodafone Group	BBB+	438,969 €	35,118 €
The Royal Bank of Scotland	A-	433,896 €	34,712 €
SoftBank	BB+	1,321,858 €	105,749 €
Sydney Airport	BBB	574,694 €	45,976 €
Pepsi	A+	29,524 €	2,362 €
Philip Morris	AA	4,412 €	353 €
Berlin, Land	AA+	4,016 €	321 €
Total	-	4,708,365 €	376,669 €

Then we summarize the all results (capital requirement) from different agreements and approaches in *Figure 4.1*. In this figure, we can see the capital requirement by using foundation internal rating-based approach of Basel II and Basel III are lower than other approach. The highest capital requirement is under Basel III—FIRB and the result is 693,000€. Comparing Basel III—SA and Basel III--FIRB, the absolute change of risk weighted asset of different approach under Basel III is 1,891,635€ and the relative change of capital requirement of different approach under Basel III is -46.65. We can see the value of capital requirements under Basel I, Basel II—SA and Basel III—SA are closed and the relative changes of three Basel agreements are respectively 3.13% and 31.25%. It means the approach of three Basel agreements is similar and the results are closed. Then we compare Basel II—FIRB and Basel III—FIRB, the relative change of two agreements is 16.03%. Using foundation internal rating-based approach relays on the internal information of banks or corporates and the results are more closed to their actual situation.

Figure 4.1 Capital requirement under Basel



4.3 Calculate the credit risk by CreditMetrics^{TM}

In this part, we use CreditMetrics^{TM} model to calculate the credit risk. Firstly, we consider the yields derived from the combination of the correlation matrix and covariance matrix. In second step is to estimate the value of each bonds for each rating and determines the forward yields curves to obtain the transition matrix. Then, do the Monte Carlo simulation. We input 25,000 random yields for each bond. Using value of correlated returns to obtain the sum of the random yields and the Cholesky decomposition matrix. Each individual yield is based on transition between the specified rating categories. In order to obtain the value of overall portfolio, we can add all the value of each individual bonds. At the end, using the risk characteristics to calculate the results.

4.3.1 The correlation among bonds

The correlation can be obtained by using market prices of each shares. The time horizon of the value of the shares in our calculation is the period from 14th, March, 2016 to 10th, March, 2017 can be shown in Annex 2. *Tab 4.8* presents the correlations among individual issuers.

Tab 4.8 Correlation among individual issuers

	Toyota Motor	Renault	Nokia	Vodafone Group	The Royal Bank of Scotland	SoftBank	Sydney Airport	Pepsi	Philip Morris	Berlin, Land
Toyota Motor	1.000	0.055	-0.006	0.523	0.331	0.027	0.327	0.266	0.160	0.265
Renault	0.055	1.000	0.225	0.280	0.189	0.279	0.250	0.301	0.300	0.071
Nokia	-0.006	0.225	1.000	0.329	0.167	0.497	0.265	0.409	0.451	0.214
Vodafone Group	0.523	0.280	0.329	1.000	0.333	0.425	0.514	0.773	0.706	0.342
The Royal Bank of Scotland	0.331	0.189	0.167	0.333	1.000	0.264	0.293	0.274	0.250	0.033
SoftBank	0.027	0.279	0.497	0.425	0.264	1.000	0.408	0.533	0.566	0.223
Sydney Airport	0.327	0.250	0.265	0.514	0.293	0.408	1.000	0.472	0.436	0.261
Pepsi	0.266	0.301	0.409	0.773	0.274	0.533	0.472	1.000	0.952	0.322
Philip Morris	0.160	0.300	0.451	0.706	0.250	0.566	0.436	0.952	1.000	0.273
Berlin, Land	0.265	0.071	0.214	0.342	0.033	0.223	0.261	0.322	0.273	1.000

In this step, correlation matrix and covariance matrix are determined by the yields of each individual shares. We can use Excel tool– Data – Data Analysis to obtain these two matrixes. The results of covariance matrix can be found in Annex 3. In *Tab 4.8*, it shows the relationship between each bond. Higher correlations between two bonds means same or higher relationship with the industry. Lower correlations between two bonds indicate same or lower relationship with the industry. For example, the value of correlation between Pepsi and The royal bank of Scotland is 0.25 as shown in *Tab 4.8* because these two industries have less relationship.

4.3.2 Calculation of the value of bonds

In the second step, the present value of the bonds should be calculated. It is necessary to create a multiannual transition matrix and the risk-free rate, probability of default and recovery rate. Using the annual transition matrix to compute the multiannual transition matrix which is presented in Annex 1. The multiannual transition matrixes can be found in Annex 4. Each bond's probability of default is the last column of each multiannual transition matrix. The recovery rate we can find in Chapter 2, Tab 2.4 and the recovery rate is 51.13%. The value of risk-free rate can be derived from the interest rate swap (IRS) during 2017 to 2026, which can be found on website of Erste Group. Based on (3.23), we can calculate the forward rates. *Tab 4.9* presents the spot rates and the forward rates from 2017 to 2026.

Tab 4.9 Spot rate (IRS) and the forward rates from 2017 to 2026

Year	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
	1	2	3	4	5	6	7	8	9	10
IRS	-0.20%	-0.11%	0.03%	0.16%	0.29%	0.42%	0.54%	0.66%	0.78%	0.88%
Forward rate	-0.20%	-0.02%	0.30%	0.55%	0.83%	1.05%	1.28%	1.50%	1.73%	1.85%

Source: Erste Group

Next, it is necessary to use (3.27) to calculate the yield curve of bonds, considering all maturities and credit ratings. The results of the forward yield from 2017 to 2026 can be found in Annex 5.

After that, the present value of all bonds should be determined by using (3.19). The values are shown in *Tab 4.10*.

Tab 4.10 Present value of bonds (€)

Bond	Toyota Motor	Renault	Nokia	Vodafone Group	The Royal Bank of Scotland	SoftBank	Sydney Airport	Pepsi	Philip Morris	Berlin, Land
AAA	1,076	1,073	59,657	126,946	59,841	273,760	109,831	104,585	104,788	949
AA+	1,076	1,073	59,654	126,927	59,836	273,681	109,802	104,572	104,779	949
AA	1,076	1,073	59,654	126,926	59,836	273,675	109,800	104,572	104,779	949
AA-	1,075	1,073	59,650	126,904	59,831	273,590	109,768	104,557	104,769	948
A+	1,075	1,073	59,639	126,853	59,815	273,429	109,705	104,519	104,740	948
A	1,072	1,072	59,555	126,609	59,719	272,798	109,453	104,327	104,567	945
A-	1,073	1,072	59,586	126,650	59,748	272,808	109,462	104,369	104,617	945
BBB+	1,070	1,071	59,512	126,364	59,651	272,004	109,140	104,153	104,440	941
BBB	1,067	1,070	59,453	126,119	59,570	271,318	108,862	103,968	104,292	939
BBB-	1,062	1,067	59,243	125,515	59,330	269,818	108,256	103,489	103,859	933
BB+	1,062	1,070	59,353	125,597	59,416	269,804	108,246	103,583	104,003	932
BB	1,050	1,062	58,889	124,328	58,892	266,832	107,033	102,561	103,062	921
BB-	1,020	1,049	57,941	121,204	57,730	258,975	103,824	100,117	100,954	890
B+	1,000	1,038	57,098	118,909	56,750	254,064	101,775	98,206	99,177	873
B	956	1,002	54,883	113,747	54,380	243,173	97,310	93,879	94,945	835
B-	818	896	48,178	97,866	47,189	208,784	83,267	80,655	82,094	712
CCC	659	697	37,912	78,483	37,480	168,206	67,219	64,615	65,327	577
D	511	511	25,565	51,130	25,565	102,260	51,130	51,130	51,130	511

From the *Tab.4.10,* the colorful cells present the default values which the bonds with the assigned credit rating by credit agency. For example, the default value of Toyota Motor is

1,076€, the default value of Renault is 1,067€ and so on. Also, the values of bonds of default can be calculated by multiplying the recovery rate and the size of exposure.

4.3.3 Simulation of the value of the portfolio

In third step, in order to do the Monte Carlo simulation, we need to generate the random yields. Using Excel tool – Data – Data Analysis – Random Number Generator to obtain the random yields. Considering normal distribution N（0,1）and the number of random yields is 25,000 of each bond. The scenarios are shown in Annex 6.

Now due to the individual bonds are independent, it should consider the dependency if the yields are simulated. We can use Cholesky decomposition matrix to obtain these results which is listed in *Tab 4.11*.

Tab 4.11 Cholesky decomposition matrix

	Toyota Motor	Renault	Nokia	Vodafone Group	The Royal Bank of Scotland	SoftBank	Sydney Airport	Pepsi	Philip Morris	Berlin, Land
Toyota Motor	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Renault	0.055	0.998	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Nokia	-0.006	0.225	0.974	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Vodafone Group	0.523	0.251	0.283	0.764	0.000	0.000	0.000	0.000	0.000	0.000
The Royal Bank of Scotland	0.331	0.171	0.134	0.103	0.912	0.000	0.000	0.000	0.000	0.000
SoftBank	0.027	0.278	0.446	0.282	0.130	0.792	0.000	0.000	0.000	0.000
Sydney Airport	0.327	0.232	0.220	0.291	0.093	0.180	0.815	0.000	0.000	0.000
Pepsi	0.266	0.287	0.355	0.603	0.030	0.144	0.044	0.577	0.000	0.000
Philip Morris	0.160	0.292	0.397	0.573	0.038	0.173	0.034	0.541	0.280	0.000
Berlin, Land	0.265	0.057	0.208	0.170	-0.120	0.094	0.073	0.079	-0.080	0.902

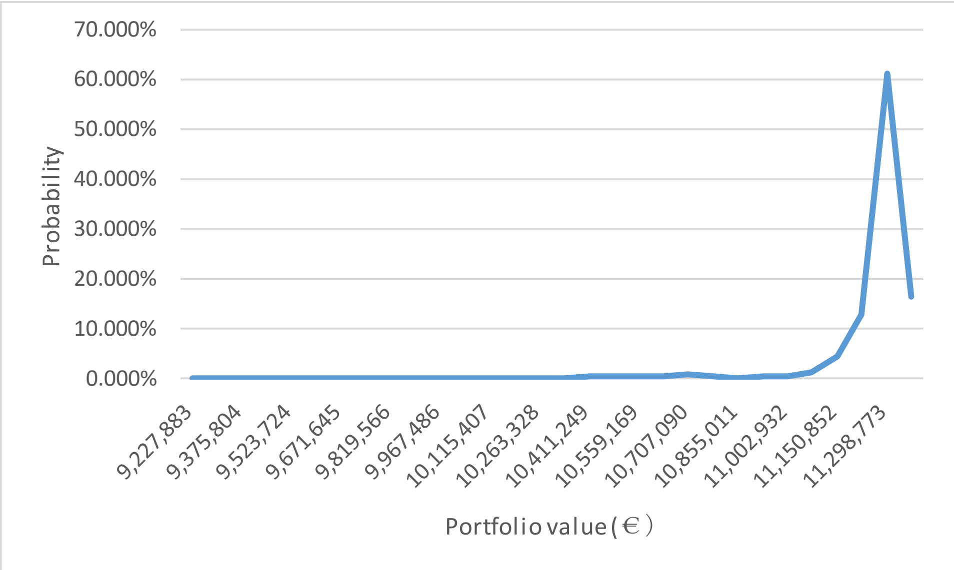
Each element of the Cholesky decomposition matrix can use (3.45), (3.46) and (3.47). Using this matrix and the random variables of standard normal distribution can compute the correlated random variables. The results of correlated random variables are presented in Annex 7.

Then, it is necessary to assign a credit rating to each correlated yield. In the previous subchapter 3.2.2, we can obtain the limit of the transition which is shown in Annex 8. Moreover, using Excel function – IF to compute the individual correlated yield of the default ratings and the results can be found in Annex 9. Due to knowing the assigned rating and present value of bonds (*Tab 4.10*), using Excel function – IF to check the values of each bond. Multiplying these values and the number of pieces of bonds to generate the total value of bonds, which are presented in Annex 10. The value of overall portfolio can obtain due to summing up the total value of bonds.

4.3.4 Calculation of credit risk

In the fourth step, it is necessary to estimate the credit risk of the portfolio. *Figure 4.2* indicates the probability distribution of the value of portfolio. In this figure, the range of portfolio value is from 9,227,883€ to 11,298,773€ and the probability is 61.235%. The results (default values) of this figure are shown in Annex 11.

Figure 4.2 Probability distribution of the portfolio values



Then the value of each bond with initial ratings and expected values are listed in *Tab 4.12*. The expected loss is the difference between the value at initial rating and expected value.

Tab 4.12 Value of portfolio (€)

	Value at initial rating	**Expected value**	**Expected loss**
Toyota Motor	1,075,814	1,075,480	334
Renault	1,067,254	1,055,593	11,661
Nokia	1,187,058	1,175,023	12,035
Vodafone Group	1,263,641	1,214,540	49,100
The Royal Bank of Scotland	1,194,958	1,194,577	381
SoftBank	1,349,021	1,343,357	5,664
Sydney Airport	1,088,624	1,089,331	-707
Pepsi	1,045,193	1,044,557	635
Philip Morris	1,047,792	1,047,740	53
Berlin, Land	948,669	948,384	284
Portfolio	11,268,024	11,188,582	79,441

In this table, we can find the expected loss of the whole portfolio is 79,441€ because of the high quality of the bonds in the portfolio. The highest expected loss is Vodafone Group and the value is 49,100€ (accounting for 61.81% of the whole expected loss) because Vodafone has highly correlation (for example, the correlation between VG and Toyota Motor is 0.523 and teh correlation between VG and Pepsi is 0.773) with other companies and the stock prices are floating. For other bonds, the expected loss of Sydney Airport is negative because the expected value is more higher than the initial value.

Next step is to present the parameter of risk – standard deviation. It indicates the scatter of values of the mean value. Also, the marginal standard deviation should be considered in the calculation because it can help to know which assets should be in the portfolio and it is possible to analyze the relationship of each bond in the whole risk. The two parameters of risks are shown in *Tab 4.13*.

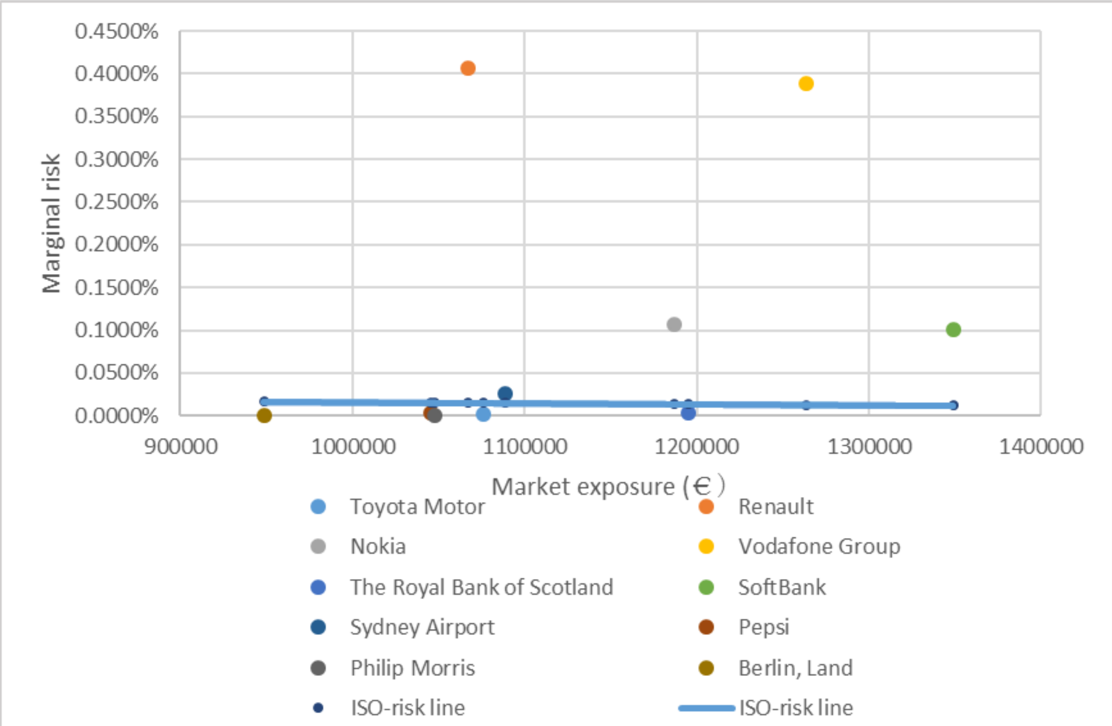
Tab 4.13 Two parameters of risk

	Standard deviation		Marginal standard deviation	
	%		%	
Toyota Motor	0.053%	575	0.001%	128
Renault	7.942%	83,834	0.407%	45,009
Nokia	2.292%	26,934	0.107%	11,832
Vodafone Group	6.573%	79,827	0.389%	43,033
The Royal Bank of Scotland	0.222%	2,655	0.003%	357
SoftBank	2.124%	28,528	0.100%	11,094
Sydney Airport	1.459%	15,889	0.026%	2,888
Pepsi	0.126%	1,315	0.003%	351
Philip Morris	0.005%	53	0.000%	9
Berlin, Land	0.159%	1,507	0.001%	61
Portfolio	1.369%	151,293		

From Tab 4.13, the standard deviation of portfolio is 1.369% and it is relatively low because the portfolio has higher risk transformation. It is easily to find the riskiest bond is Renault with standard deviation 7.942% because it has the lowest initial credit rating in the portfolio. Moreover, the marginal standard deviation of Renault is the highest with 0.407%. The big difference between standard deviation and marginal standard deviation is the diversification. Lower marginal standard deviation means a good effect of diversification. We can find the marginal standard deviation of Toyota Motor, Philip Morris and Berlin, Land are lower, whose are 0.001%, 0.000% and 0.001% because their initial rating are higher.

Figure 4.4 below presents the marginal risk of all bonds and ISO-risk line. ISO-risk line includes points with the same results of the absolute marginal risk. The absolute marginal risk is equal to the market exposure times marginal standard deviation. ISO-risk line shows if the bond locates above the line is much risky than bond locates below the line.

Figure 4.4 Marginal risks and market exposure



It is simple to see there are four bonds are above the ISO-risk line, which are Renault, Vodafone Group, Nokia and Softbank. These points mean these four bonds are more risky to invest. Others are below or on the line.

The next important thing is to calculate the economic capital. It is necessary to know the significant level and it is usually fixed at 0.1%, 0.5% and 1%. There are two steps to obtain the economic capital. Firstly, *Tab 4.14* presents the portfolio value and unexpected losses (VaR) at three significant levels.

Tab 4.14 Significant level and corresponding value of the portfolio and losses (€)

alpha	Portfolio value	VaR
0.1%	9,804,133	-1,463,890
0.5%	10,352,165	-915,859
1%	10,467,689	-800,334

In this table, the portfolio value and VaR are 9,804,133 € and 1,463,890 € at the significant level 0.1% (also notes confidence level at 99.9%). Secondly, it can be used to calculate the economic capital. We can use (3.18) to calculate the economic capital and the results are presents in *Tab 4.15*.

Tab 4.15 Significate level and economic capitals (€)

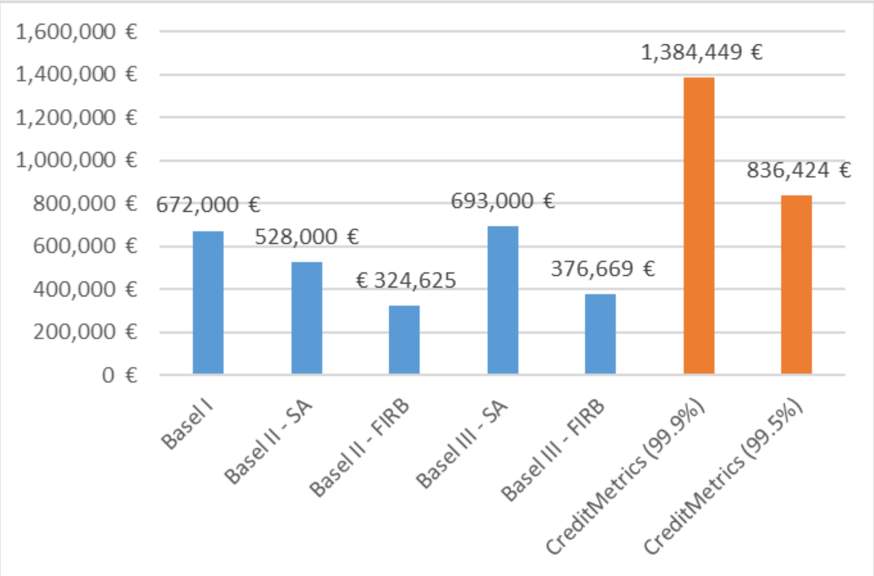
alpha	Economic capital
0.1%	1,384,449
0.5%	836,424
1%	720,896

The economic capital at a confidence level 99.9% is 1,384,449€, while 836,424€ at a confidence level 99.5%. It is a big change between confidence level at 99.9% and 99.5% because of the heavy effect of credit risk.

4.4 Evaluation of results

In the subchapter 4.2 we use Basel agreements to calculate the capital requirement to cover unexpected losses. In subchapter 4.3, the economic capital is computed by CreditMetrics^{TM} model. Then in this subchapter, we summarize all results and analyze. The results can be found in *Figure 4.5*.

Figure 4.5 Capital requirement under different methods



In the figure above, the economic capital by the CreditMetrics^{TM} model at confidence level 99.9% is the highest and the credit risk is the highest in the portfolio of six methods. Moreover, when the confidence level is 99.5%, the economic capital is closed to the capital requirement calculated by standard approach under Basel III. It is easy to find the value of capital requirement is lower when calculated by foundation internal rating-based approach than used by standard approach in Basel II and Basel III. For example, we can see the capital requirements under Basel I, Basel II and Basel III—SA are closed and the relative changes of three Basel agreements are respectively 3.13% and 31.25%. It means the approach of three Basel agreements is similar and the results are closed. When compares Basel II—FIRB and Basel III—FIRB, the relative change of two agreements is 16.03%.

Then we can explain some reasons why different values of capital requirement by used different methods.

One reason is the typical granularity of the diversification and concentration of the portfolio. It can be fouced on Basel agreements, especially the foundation internal rating-based approach. Higher granularity is preferable because it can help borrowers to find more diverse rating of the securities and it can make the loan's price more stable. Using foundation internal rating-based approach can be more closed to the accurate situation of the banks, corporates. In addition, it helps to estimate the risk and calculate the regulatory capital requirements. Investors can learn from these information to invest. Second is different degree of correlation in different methods. Higher correlation indicates higher level of capital requirement. We can find the correlation (R, it can be determined by probability of default) of Basel II—FIRB and Basel III—FIRB is from 20% to 24%, which is relatively narrow. While, in $\text{CreditMetrics}^{\text{TM}}$ model, the range of correlation can be from -0.6% to 95% and it is more widely. Third reason is using different significant level. Therefore, the values are different of capital requirement when choosing different methods.

5 Conclusion

Nowadays, financial risk runs through all aspects of our lives, so risk management becomes more important. For the bank, the largest financial risk which the most banks face is the credit risk. Credit risk is the potential loss that the bank borrower fails to fulfill its obligations, then the bank borrower pays the loan interest and repays the loan amount under the agreed terms. Therefore, calculate the credit risk of banking sector is the necessary thing.

The aim of this thesis is to compute and compare the value of capital requirement for unexpected losses based on credit risk of ten debt assets portfolio under Basle I, Basel II and Basel III. Also, we use CreditMetrics™ model to calculate the economic capital.

The thesis included three main parts, description, mathematical analysis and practical part. The description part mainly introduced and described the several banks' risks and the credit risk management and models. At first, some important terms and equations of bank's risks were introduced. In the mathematical analysis part (Chapter 3), most parts described the CreditMetrics™ model for calculating the economic capital of credit risk. Basel agreements (includes Basel I, Basel II and Basel III) were described in last part of Chapter 3. In the practical part, we selected ten debt assets with a portfolio traded on Frankfurt Stock Exchange (FSE) to act as the basic data of the thesis. The nominal value of one debt asset was 1 million euro (hence the nominal value of overall portfolio was 10 million euro). The time periods of the credit risk determined one year. Then was the calculation of the portfolio. First, we used the Basel agreements include the standard approach (SA) and foundation internal ratings-based approach (FIRB) to calculate the value of capital requirement of unexpected losses. Next, the calculation of economic capital by CreditMetrics™ model was presented.

The results were presented in Chapter 4. We used two main methods to compare the credit risk. One was the Basel agreements to calculate the capital requirement to cover the unexpected losses. Another was the CreditMetrics™ model to calculate the economic capital. The value of economic capital calculated by using CreditMetrics™ model at the confidence level of 99.5% was 875,941€, the result was closed to the value of capital requirement calculated based on Basel III by the foundation internal rating-based approach which was 693,000€. The reason of different results can be the different degree of correlation of different methods. We could find the difference of range of the correlation which was determined by probability of default between Basel II and Basel III by foundation internal rating-based approach was 2% (from 20% to 24%). The range of correlation based on credit matrix by using

$\text{CreditMetrics}^{\text{TM}}$ was wide (from -0.6% to 95%). In addition, we compared the results of Basel agreements. The value of capital requirement computed by standard approach under Basel II and Basel III was higher than the value of capital requirement calculated by foundation internal rating-based approach under Basel II and Basel III. The relative changes between the value of capital requirement computed by standard approach and foundation internal rating-based approach under Basel II was approximately 38.52% and the relative changes between the value of capital requirement computed by standard approach and foundation internal rating-based approach under Basel III was 45.65%.

At the end, risk management is always necessary in our life. The process of risk management will be better developed in the future because of the needs of people, the improvement of science and technology and the progress of society. For the credit risk, banks and governments should also strengthen regulation and cooperation to reduce the risk.

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List of Abbreviations

CAR	Capital adequancy ratio
CR	Capital requirement
DD	Distance of default
DP	Default point
EAD	Exposure at default
EC	Economic capital
EDF	Expected default frequency
EL	Expected loss
FIRB	Foundation Internal Rating-based approach
FSE	Frankfurt Stock Exchange
IRS	Interest rate swap
LCR	Liquidity coverage ratio
LGD	Loss given default
LLA	Loan loss allowance
LTD	Long-term debt
NPL	Net performing loans
OECD	Organization for Economic Co-operation and Development
OTC	Over-the-counter market
PD	Probability of default
ROC	Receiver operating characteristic
RR	Recovery rate
RWA	Risk-weighted asset
SA	Standard Approach
STD	Short-term debt
UL	Unexpected loss
VaR	Value at risk
VG	Vodafone Group

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Annex 1: Probability matrix from Standard & Poor's

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	85.03%	6.72%	1.52%	0.87%	0.22%	0.43%	0.00%	0.00%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA+	1.09%	74.86%	15.03%	2.73%	0.82%	0.82%	0.55%	0.55%	0.00%	0.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.22%	1.20%	78.98%	8.50%	4.14%	1.31%	0.54%	0.22%	0.00%	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA-	0.08%	0.08%	4.56%	74.98%	12.26%	2.73%	1.24%	0.17%	0.08%	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A+	0.00%	0.07%	0.63%	5.51%	73.97%	10.89%	2.58%	0.49%	0.35%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A	0.00%	0.23%	0.17%	0.74%	4.69%	73.46%	11.21%	2.29%	1.14%	0.17%	0.06%	0.06%	0.06%	0.06%	0.00%	0.00%	0.00%	0.11%
A-	0.05%	0.00%	0.16%	0.16%	0.98%	7.22%	76.11%	7.93%	1.48%	0.82%	0.16%	0.05%	0.11%	0.00%	0.00%	0.00%	0.00%	0.05%
BBB+	0.00%	0.00%	0.00%	0.14%	0.29%	0.86%	7.43%	73.50%	8.71%	1.21%	0.36%	0.57%	0.21%	0.21%	0.07%	0.00%	0.14%	0.07%
BBB	0.00%	0.00%	0.10%	0.00%	0.19%	0.58%	0.88%	7.89%	69.98%	7.89%	1.66%	1.07%	0.10%	0.10%	0.39%	0.10%	0.10%	0.10%
BBB-	0.00%	0.00%	0.16%	0.00%	0.16%	0.64%	0.48%	1.43%	8.90%	67.25%	6.52%	2.70%	0.79%	0.32%	0.32%	0.00%	0.32%	0.32%
BB+	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.30%	0.60%	0.90%	11.64%	58.81%	8.06%	2.39%	1.79%	0.30%	0.00%	0.30%	0.00%
BB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.50%	0.00%	1.75%	11.25%	56.75%	6.25%	2.75%	1.00%	0.00%	0.75%	0.50%
BB-	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.25%	0.25%	8.89%	59.01%	12.84%	4.20%	0.49%	0.25%	1.48%
B+	0.00%	0.00%	0.00%	0.00%	0.00%	0.23%	0.00%	0.23%	0.00%	0.00%	0.23%	2.93%	8.80%	54.63%	8.35%	3.84%	1.35%	1.81%
B	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.38%	0.38%	0.38%	1.51%	12.08%	45.66%	8.30%	4.53%	4.15%
B-	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.63%	0.00%	0.00%	0.00%	0.00%	1.27%	6.33%	49.37%	15.82%	10.13%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.15%	3.46%	9.20%	25.29%	37.93%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Annex 2: Shares prices from March 14th, 2016 to March 10th, 2017 (€)

Date	**Toyota Motor**	**Renault**	**Nokia**	**Vodafone Group**	**The Royal Bank of Scotland**	**SoftBank**	**Sydney Airport**	**Pepsi**	**Philip Morris**	**Berlin, Land**
14/03/2016	110.56	107.65	116.01	101.03	116.15	101.81	106.85	106.28	105.74	105.12
15/03/2016	111.27	107.58	115.89	101.13	116.45	101.72	107.52	106.09	105.62	105.16
16/03/2016	111.17	107.58	115.89	101.02	116.3	101.62	107.49	106.01	105.51	104.92
17/03/2016	111.21	107.6	115.89	101.46	116.28	101.75	107.04	106.09	105.56	104.83
18/03/2016	111.54	107.62	115.77	101.81	116.22	102	107.5	106.21	105.61	105.5
21/03/2016	111.42	107.63	115.89	101.96	116.2	102.3	107.86	106.26	105.57	105.88
22/03/2016	111.6	107.61	115.89	102.01	116.21	102.59	107.58	106.31	105.58	105.67
23/03/2016	111.58	107.71	115.89	101.94	116.18	102.81	107.78	106.19	105.5	105.54
24/03/2016	111.8	107.69	115.94	102.52	116.2	102.8	108.38	106.41	105.61	106
29/03/2016	112	107.71	115.89	102.8	116.18	102.81	108.13	106.52	105.68	106
30/03/2016	112.2	107.7	115.89	102.93	116.24	102.85	108.61	106.5	105.71	106.04
31/03/2016	112.12	107.66	115.72	102.98	116.25	103.14	108.61	106.52	105.68	106.35
01/04/2016	112.03	107.7	115.89	103.1	116.28	103	108.66	106.53	105.65	106.13
04/04/2016	112.34	107.76	115.89	103.76	116.29	103.48	108.79	106.56	105.64	106.08
05/04/2016	112.4	107.79	115.76	104.28	116.36	103.25	109.01	106.82	105.83	106.27
06/04/2016	112.46	107.78	115.76	104.34	116.36	103.23	108.7	106.81	105.79	106.47
07/04/2016	112.57	107.76	115.76	104.27	116.37	103.31	108.92	106.89	105.81	106.64
08/04/2016	112.78	107.77	115.76	104.37	116.37	103	108.97	106.93	105.87	106.39
11/04/2016	112.79	107.76	115.77	104.57	116.38	102.84	108.88	106.98	105.82	106.57
12/04/2016	112.72	107.75	115.77	104.14	116.29	102.83	108.56	106.93	105.82	106.57
13/04/2016	112.37	107.74	115.78	104.04	115.88	102.93	108.42	106.86	105.78	106.19
14/04/2016	112.54	107.67	115.76	104.17	115.75	103.19	108.46	106.83	105.79	105.95
15/04/2016	112.34	107.68	115.68	103.95	115.64	103	108.53	106.83	105.69	106.25

18/04/2016	112.59	107.65	115.64	104.29	115.61	102.95	108.54	106.86	105.73	106.06
19/04/2016	112.39	107.65	115.66	103.82	115.61	103.3	108.45	106.79	105.72	106.35
20/04/2016	112.46	107.64	115.68	103.99	115.6	103.61	108.67	106.85	105.73	106.35
21/04/2016	112.27	107.57	115.66	103.86	115.69	103.71	108.56	106.7	105.58	106.17
22/04/2016	112.06	107.64	115.64	103.35	115.78	103.9	108.71	106.5	105.38	106.17
25/04/2016	112.22	107.65	115.69	103.71	115.93	104.17	108.77	106.78	105.58	105.41
26/04/2016	111.98	107.61	115.66	103.43	115.93	104.12	108.51	106.65	105.52	105.4
27/04/2016	111.71	107.6	115.67	103.23	115.9	104.47	108.28	106.54	105.41	105.18
28/04/2016	112.02	107.59	115.66	103.69	115.92	104.6	108.63	106.71	105.51	104.77
29/04/2016	111.96	107.55	115.64	103.6	115.93	104.74	108.55	106.67	105.5	105.27
02/05/2016	111.58	107.56	115.47	103.34	115.82	104.95	108.63	106.4	105.3	105.18
03/05/2016	111.52	107.56	115.57	103.27	115.74	104.48	108.63	106.35	105.25	105
04/05/2016	111.86	107.58	115.65	103.59	115.9	104.48	109.03	106.52	105.29	105
05/05/2016	112.07	107.58	115.64	103.63	115.84	104.5	109.09	106.53	105.24	105.49
06/05/2016	112.3	107.59	115.64	104.15	115.88	104.55	109.4	106.69	105.38	105.49
09/05/2016	112.19	107.59	115.64	104.44	115.87	104.86	109.27	106.88	105.56	106.09
10/05/2016	112.31	107.54	115.64	104.47	115.86	104.68	109.17	106.85	105.52	106.02
11/05/2016	112.34	107.54	115.64	104.39	115.87	104.98	109.27	106.71	105.43	106.25
12/05/2016	112.33	107.51	115.64	104.27	115.86	105.15	109.34	106.71	105.43	106.41
13/05/2016	112.17	107.45	115.6	104.05	115.82	105.19	109.13	106.6	105.38	106.45
17/05/2016	112.13	107.43	115.58	103.66	115.78	105.38	109.12	106.31	105.25	106.28
18/05/2016	111.86	107.41	115.57	104.03	115.78	105.71	109.25	106.51	105.49	106
19/05/2016	111.66	107.38	115.58	103.67	115.71	106	108.82	106.47	105.36	105.98
20/05/2016	111.68	107.37	115.52	103.69	115.71	105.72	108.98	106.39	105.35	105.7
23/05/2016	111.72	107.36	115.54	103.72	115.71	105.94	108.73	106.42	105.24	105.95
24/05/2016	111.59	107.36	115.5	103.57	115.67	106.22	108.88	106.32	105.19	106.07

25/05/2016	111.64	107.36	115.5	103.48	115.76	106.59	108.83	106.31	105.26	106.14
26/05/2016	111.77	107.4	115.53	103.82	115.79	106.52	109.05	106.4	105.27	106.14
27/05/2016	111.86	107.41	115.41	104.04	115.8	106.62	109.15	106.48	105.32	106.14
30/05/2016	111.99	107.4	115.37	104.1	115.82	106.62	108.95	106.45	105.26	106.14
31/05/2016	111.76	107.43	115.6	103.88	115.77	106.54	109	106.34	105.18	106.14
01/06/2016	112.01	107.46	115.39	104.28	115.8	107.24	109.25	106.5	105.31	106.14
02/06/2016	112.03	107.45	115.41	104.26	115.74	107.14	109.29	106.51	105.29	106.28
03/06/2016	112.14	107.55	115.39	104.57	115.72	107.46	109.58	106.56	105.33	106.28
06/06/2016	112.44	107.58	115.51	104.88	115.77	107.17	109.64	106.65	105.36	106.3
07/06/2016	112.35	107.59	115.56	104.8	115.69	107.13	109.77	106.67	105.39	106.51
08/06/2016	112.51	107.71	115.46	105.17	115.68	107.21	109.91	106.78	105.43	106.78
09/06/2016	112.58	107.68	115.44	105.26	115.66	106.96	110.13	106.76	105.49	106.69
10/06/2016	112.63	107.67	115.46	105.34	115.66	106.88	110.56	106.79	105.55	106.95
13/06/2016	112.72	107.71	115.47	105.29	115.65	106.33	110.53	106.85	105.48	106.95
14/06/2016	112.68	107.74	115.44	105.46	115.59	105.56	110.59	106.91	105.53	107.02
15/06/2016	112.84	107.67	115.44	105.26	115.64	105.2	110.65	106.99	105.56	106.99
16/06/2016	112.93	107.76	115.4	105.3	115.6	105	110.85	106.95	105.61	106.99
17/06/2016	112.9	107.74	115.39	105.21	115.42	105.22	110.61	106.95	105.57	107.28
20/06/2016	112.68	107.71	115.35	104.79	115.44	105.68	110.61	106.98	105.5	107.07
21/06/2016	112.6	107.71	115.35	104.82	115.41	106.3	110.61	106.95	105.52	107.07
22/06/2016	112.61	107.77	115.33	104.87	115.47	106.2	110.45	106.94	105.51	106.82
23/06/2016	112.64	107.76	115.25	104.83	115.45	106.41	110.54	106.93	105.58	106.88
24/06/2016	113.04	107.51	113.9	103.51	114.84	101.7	109.42	105.46	103.85	106.62
27/06/2016	112.89	107.66	115.15	104.85	114.72	104.61	110.64	106.87	105.4	107.49
28/06/2016	112.95	107.67	115.14	104.83	114.22	105.05	110.81	106.92	105.46	107.54
29/06/2016	113.21	107.68	115.16	105.07	114.23	105.39	111.13	106.92	105.54	107.75
30/06/2016	113.14	107.6	115.04	105.15	114.24	105.47	111.54	106.96	105.52	107.89

01/07/2016	113.01	107.65	115.11	105.7	113.89	105.64	111.54	107.02	105.69	107.83
04/07/2016	113.44	107.75	115.17	105.85	113.99	105.79	111.86	107.26	105.77	107.83
05/07/2016	113.76	107.79	115.19	106.23	114.2	105.99	111.87	107.46	105.88	108.15
06/07/2016	114.06	107.86	115.17	106.56	114.15	106.1	112.18	107.6	106.06	108.72
07/07/2016	114.11	107.83	115.2	106.54	114.06	106.48	112.38	107.64	106.08	108.99
08/07/2016	114.12	107.91	115.18	106.68	113.95	106.91	112.71	107.74	106.17	108.98
11/07/2016	114.14	107.93	115.16	107.07	114	106.97	112.74	107.86	106.28	108.98
12/07/2016	114.06	107.9	115.16	106.81	113.96	107.33	112.52	107.86	106.29	109.14
13/07/2016	114.03	108.01	115.27	107.2	114.5	107.93	112.75	107.85	106.32	108.58
14/07/2016	114.21	108.09	115.27	107.65	114.68	108.22	112.79	107.94	106.43	108.53
15/07/2016	114.25	108.04	115.16	107.98	114.76	108.22	112.38	107.83	106.33	108.67
18/07/2016	114.1	107.94	115.25	107.91	114.75	106.47	112.73	107.84	106.34	108.6
19/07/2016	114.39	107.96	115.28	108.41	114.78	106.22	113.05	107.91	106.38	108.33
20/07/2016	114.37	107.91	115.28	108.56	114.89	106.48	113.07	107.96	106.39	108.65
21/07/2016	114.22	107.9	115.27	108.24	114.81	107.67	112.85	107.78	106.25	108.37
22/07/2016	114.23	107.88	115.24	108.48	114.85	107.23	112.97	107.84	106.28	108.14
25/07/2016	114.07	107.85	115.25	108.7	114.83	107.34	113.04	107.87	106.28	108.1
26/07/2016	114.17	107.87	115.24	109.05	114.78	107.52	113.05	107.94	106.31	108.11
27/07/2016	114.11	107.88	115.27	108.94	114.74	107.51	113.25	107.87	106.29	108.4
28/07/2016	114.32	107.8	115.77	109.23	114.8	108.21	113.56	107.89	106.31	108.18
29/07/2016	114.31	107.84	115.64	109.22	114.7	108	113.47	107.86	106.29	108.82
01/08/2016	114.4	107.8	115.64	109.46	114.78	108.39	113.57	107.81	106.33	108.66
02/08/2016	114.25	107.73	115.59	109.17	114.74	108.66	113.04	107.75	106.26	108.93
03/08/2016	114.05	107.63	115.59	108.87	114.7	109.4	113.2	107.77	106.18	108.7
04/08/2016	114.05	107.63	115.73	108.88	114.79	109.12	113.31	107.79	106.18	108.4
05/08/2016	114.36	107.67	115.76	109.4	114.95	109.78	113.69	107.92	106.3	108.54
08/08/2016	114.13	107.67	115.71	109.34	114.91	110	113.56	107.88	106.27	108.94

09/08/2016	114.29	107.62	115.65	109.44	114.98	110.21	113.74	107.92	106.27	108.51
10/08/2016	114.29	107.52	115.58	109.79	115.05	110.72	113.92	107.94	106.3	108.76
11/08/2016	114.69	107.59	115.57	110	115.09	110.73	114.17	108.01	106.35	109.05
12/08/2016	114.41	107.57	115.5	109.8	115.22	110.87	114.28	107.91	106.26	109.24
15/08/2016	114.49	107.52	115.29	109.9	115.3	111.21	114.12	107.88	106.24	108.89
16/08/2016	114.29	107.5	115.29	109.79	115.21	111.69	114.1	107.87	106.19	109.05
17/08/2016	114.13	107.56	115.3	109.52	115.23	111.93	113.89	107.82	106.16	108.9
18/08/2016	114.18	107.39	115.27	109.61	115.23	111.65	114.13	107.83	106.16	108.56
19/08/2016	114.35	107.42	115.27	109.82	115.27	111.72	114	107.88	106.16	108.86
22/08/2016	114.1	107.43	115.29	109.53	115.15	111.49	114.05	107.8	106.15	108.94
23/08/2016	114.34	107.41	115.32	109.79	115.23	111.73	114.1	107.88	106.2	108.61
24/08/2016	114.31	107.42	115.27	109.84	115.27	111.72	114.27	107.83	106.18	108.98
25/08/2016	114.24	107.33	115.24	109.76	115.27	111.61	114.02	107.83	106.14	109.05
26/08/2016	114.14	107.3	115.29	109.71	115.22	111.62	113.99	107.76	106.09	109
29/08/2016	114.11	107.31	115.23	109.56	115.15	111.52	113.58	107.79	106.06	108.76
30/08/2016	114.12	107.32	115.25	109.7	115.26	111.45	113.99	107.8	106.11	108.92
31/08/2016	114.2	107.31	115.16	109.76	115.25	111.38	113.97	107.79	106.1	109.06
01/09/2016	114.09	107.28	115.1	109.49	115.19	111.3	113.89	107.67	106.07	108.78
02/09/2016	114.09	107.28	115.05	109.53	115.23	111.09	113.8	107.7	106.07	108.79
05/09/2016	114.05	107.23	115.04	109.51	115.29	111.25	113.78	107.75	106.07	108.79
06/09/2016	114.11	107.29	115.07	109.43	115.28	110.86	114.05	107.75	106.08	108.81
07/09/2016	114.42	107.25	115.04	109.96	115.47	110.61	114.16	107.94	106.15	109.31
08/09/2016	114.45	107.23	115.41	109.99	115.47	110.24	114.11	107.93	106.18	109.31
09/09/2016	114.21	107.23	115.18	109.37	115.37	109.92	113.56	107.63	105.96	108.68
12/09/2016	113.78	107.17	114.93	108.68	115.34	108.59	113.25	107.08	105.8	107.98
13/09/2016	113.76	107.15	115.27	108.83	115.3	108.77	113.2	107.36	105.81	108.05

14/09/2016	113.48	107.17	114.93	108.56	115.21	108.46	113.03	107.29	105.63	107.89
15/09/2016	113.75	107.1	114.92	108.59	115.3	108.3	113.07	107.33	105.68	107.89
16/09/2016	113.67	107.11	114.96	109.01	115.27	108.01	113.22	107.47	105.73	107.97
19/09/2016	113.79	107.09	114.93	108.9	115.27	107.96	113.22	107.44	105.72	108.17
20/09/2016	113.72	107.11	114.92	108.83	115.21	107.84	113.29	107.36	105.69	108.17
21/09/2016	113.48	107.09	114.88	108.81	115.21	107.77	113.24	107.36	105.64	108.15
22/09/2016	113.73	107.08	114.92	109.09	115.18	108.04	113.55	107.41	105.74	108.27
23/09/2016	114.1	107	114.82	109.43	115.22	108.38	113.66	107.46	105.76	108.5
26/09/2016	114.05	107.06	114.75	109.57	115.22	108.44	113.89	107.53	105.82	108.92
27/09/2016	113.85	107.12	114.8	109.41	115.25	108.23	113.85	107.56	105.89	109.03
28/09/2016	114.41	107.09	114.49	109.55	115.29	108.39	114.14	107.64	105.98	109.04
29/09/2016	114.43	106.99	114.76	109.48	115.19	108.34	113.93	107.58	105.9	109.22
30/09/2016	114.26	106.99	114.76	109.57	114.98	108.33	114.14	107.62	105.97	109.05
04/10/2016	114.12	106.98	114.72	109.14	114.81	108.33	113.78	107.51	105.87	109.28
05/10/2016	113.93	106.91	114.74	108.82	114.71	108.1	113.4	107.46	105.81	109.28
06/10/2016	113.4	106.87	114.72	108.62	114.61	108.1	113.2	107.32	105.75	108.54
07/10/2016	113.31	106.86	114.68	108.26	114.53	108.01	113.16	107.17	105.64	108.06
10/10/2016	113.25	106.85	114.66	108.26	114.47	108.1	112.96	107.12	105.63	107.78
11/10/2016	113.35	106.84	114.61	108.04	114.36	107.94	112.98	107.11	105.66	107.49
12/10/2016	113.43	106.81	114.58	108.2	114.09	107.98	112.85	107.13	105.61	107.49
13/10/2016	113	106.8	114.5	108.13	114.01	107.66	112.86	107.11	105.62	107.49
14/10/2016	113.39	106.77	114.49	107.99	113.88	107.57	112.78	107.11	105.63	107.58
17/10/2016	113.28	106.77	114.47	107.79	113.93	107.46	112.71	107.05	105.55	107.58
18/10/2016	113.35	106.78	114.39	108.1	113.99	107.43	112.86	107.17	105.61	107.27
19/10/2016	113.51	106.77	114.39	108.27	114.09	107.52	113.09	107.24	105.66	107.5
20/10/2016	113.55	106.79	114.39	108.13	114.14	107.63	112.97	107.17	105.61	107.7

21/10/2016	113.6	106.73	114.29	108.43	114.11	107.68	113.21	107.22	105.67	107.88
24/10/2016	113.56	106.72	114.31	108.44	114.08	107.67	113.17	107.2	105.6	107.88
25/10/2016	113.46	106.7	114.2	108.3	114.03	107.69	113.24	107.13	105.62	108.03
26/10/2016	113.38	106.66	114.26	108.19	113.99	107.82	112.67	107.09	105.58	107.79
27/10/2016	113.11	106.62	114.16	107.69	113.96	107.75	112.05	106.92	105.47	107.77
28/10/2016	112.68	106.61	113.84	107.09	113.86	107.59	112.01	106.69	105.32	107.26
31/10/2016	112.72	106.6	113.23	107.15	113.87	107.59	111.99	106.69	105.35	106.63
01/11/2016	112.76	106.59	113.3	106.86	113.9	107.57	112	106.64	105.3	106.62
02/11/2016	112.71	106.61	113.51	107.33	113.88	107.43	112.16	106.78	105.37	106.56
03/11/2016	112.94	106.57	113.49	107.25	113.88	107.69	112.03	106.77	105.37	106.84
04/11/2016	112.82	106.57	113.43	107.22	113.84	107.32	112.1	106.76	105.31	106.54
07/11/2016	112.9	106.56	113.46	107.18	113.92	107.29	112.17	106.72	105.27	106.58
08/11/2016	112.81	106.55	113.5	107.29	113.94	107.37	112.21	106.79	105.36	106.58
09/11/2016	112.99	106.55	113.39	106.97	113.93	106.64	111.73	106.69	105.15	106.76
10/11/2016	112.46	106.44	113.34	106.54	113.83	106.35	110.95	106.55	105.19	106.76
11/11/2016	112.25	106.48	113.34	105.87	113.82	106.31	110.86	106.28	105.06	105.86
14/11/2016	111.84	106.42	113.25	105.3	113.76	105.3	110.21	106.13	104.93	104.93
15/11/2016	111.8	106.5	113.14	105.62	113.78	104.85	110.26	106.29	105.02	104.82
16/11/2016	111.81	106.42	113.34	105.25	113.8	104.61	109.96	106.2	104.96	104.89
17/11/2016	111.38	106.4	112.74	105.15	113.78	104.3	110.19	106.25	104.97	104.9
18/11/2016	111.43	106.35	113.34	105.15	113.74	103.59	109.33	105.86	104.8	105.2
21/11/2016	111.04	106.35	113.1	104.34	113.78	103.2	109.75	105.96	104.86	104.79
22/11/2016	111.02	106.42	113.19	105.11	113.83	103.29	110.05	106.32	105.14	104.98
23/11/2016	111.38	106.43	112.41	105.33	113.97	103.13	109.87	106.44	105.25	105.19
24/11/2016	111.11	106.41	112.69	105.1	113.88	103.25	110.29	106.35	105.19	105.45
25/11/2016	111.23	106.43	112.99	105.07	113.94	103.25	110.27	106.45	105.29	105.55
28/11/2016	111.45	106.43	112.95	105.33	113.96	103.65	110.48	106.55	105.36	105.58

29/11/2016	111.51	106.42	112.74	105.41	114	103.75	110	106.58	105.35	105.58
30/11/2016	111.4	106.42	112.74	105.32	113.96	103.77	109.91	106.57	105.34	105.65
01/12/2016	111.04	106.36	112.74	104.47	113.86	103.72	109.52	106.41	105.2	105.72
02/12/2016	110.68	106.32	112.74	104.1	113.77	103.64	109.48	106.26	105.13	105.06
05/12/2016	111.08	106.2	112.74	104.24	113.78	103.73	109.12	106.28	105.14	104.93
06/12/2016	110.58	106.19	112.74	104.26	113.65	103.76	109.02	106.22	105.12	104.82
07/12/2016	110.39	106.11	112.76	104.29	113.61	103.82	109.09	106.13	105.06	104.82
08/12/2016	110.5	106.16	112.81	104.29	113.53	104.05	108.91	106.1	104.95	104.43
09/12/2016	110.77	106.21	112.78	104.29	113.65	104.08	109.31	106.34	105.17	104.55
12/12/2016	110.91	106.17	112.97	104.33	113.68	104.1	108.87	106.36	105.22	104.15
13/12/2016	110.86	106.18	112.97	103.84	113.64	104.2	109.06	106.35	105.16	103.98
14/12/2016	110.99	106.17	112.97	104.27	113.65	104.4	109.42	106.5	105.21	104.61
15/12/2016	110.77	106.21	112.97	103.01	113.67	103.91	109.22	105.01	103.68	104.23
16/12/2016	111.03	106.24	112.97	104.41	113.68	103.98	109.17	106.56	105.29	104.53
19/12/2016	111.14	106.25	112.81	104.46	113.73	104.05	109.57	106.58	105.27	104.33
20/12/2016	111.38	106.24	112.97	104.78	113.78	104.07	109.67	106.69	105.32	104.33
21/12/2016	111.21	106.24	112.95	104.93	113.7	104.14	109.62	106.63	105.31	104.82
22/12/2016	111.26	106.22	112.97	104.89	113.63	104.53	109.73	106.53	105.27	104.9
23/12/2016	111.19	106.21	112.97	104.92	113.59	104.57	109.7	106.57	105.18	104.77
27/12/2016	111.35	106.28	112.79	105.09	113.6	104.29	109.84	106.68	105.29	104.9
28/12/2016	111.53	106.17	112.72	105.47	113.68	104.76	110.06	106.8	105.42	105.22
29/12/2016	111.53	106.1	112.47	105.48	113.64	104.69	110.12	106.78	105.34	105.22
30/12/2016	111.5	105.84	112.72	105.4	113.54	104.63	110.18	106.66	105.27	105.39
02/01/2017	111.44	105.84	112.56	105.48	113.53	104.67	110.12	106.76	105.3	105.43
03/01/2017	111.53	106.09	112.97	105.48	113.54	104.58	109.59	106.76	105.3	105.58
04/01/2017	111.23	106.04	112.97	104.98	113.52	105.25	109.63	106.65	105.26	105.53
05/01/2017	111.29	105.94	112.82	104.88	113.52	105.68	109.73	106.53	105.19	105.03

06/01/2017	111.21	105.92	112.67	104.88	113.36	106.08	109.82	106.4	105.04	104.97
09/01/2017	111.01	105.91	112.67	104.7	113.34	106.04	109.53	106.4	105.09	105.16
10/01/2017	111.17	105.92	112.68	104.82	113.34	106.18	109.64	106.42	105.08	104.74
11/01/2017	111.16	105.9	112.67	104.72	113.28	106.3	109.56	106.34	105.05	105.03
12/01/2017	111.24	105.88	112.64	105.09	113.25	106.92	109.63	106.36	105.02	105.03
13/01/2017	111.19	105.79	112.63	104.95	113.2	106.93	109.85	106.36	105.02	105.51
16/01/2017	111.27	105.85	112.63	105.05	113.24	106.74	109.77	106.44	105.07	105.28
17/01/2017	111.34	105.84	112.62	105.05	113.3	107.05	109.77	106.5	105.12	105.39
18/01/2017	111.21	105.8	112.6	104.92	113.24	107.05	109.63	106.37	105.02	105.39
19/01/2017	110.88	105.75	112.59	104.44	113.24	107.13	109.62	106.24	104.92	105.17
20/01/2017	110.88	105.69	112.56	104.44	113.23	106.53	109.46	106.21	104.92	104.77
23/01/2017	110.71	105.75	112.56	104.34	113.13	106.49	109.25	106.12	104.87	104.63
24/01/2017	110.82	105.77	112.54	104.48	113.23	107.21	109.25	106.16	104.86	104.47
25/01/2017	110.54	105.78	112.54	104.05	113.17	106.74	109.1	106.01	104.8	104.62
26/01/2017	110.23	105.77	112.44	103.71	113.12	106.73	108.85	105.87	104.68	104.14
27/01/2017	110.3	105.75	112.42	103.69	113.09	106.68	108.83	105.86	104.72	103.71
30/01/2017	110.34	105.8	112.44	103.53	113.13	106.7	108.96	105.82	104.71	103.78
31/01/2017	110.46	105.82	112.42	103.66	113.12	106.65	109.19	105.88	104.79	103.65
01/02/2017	110.43	105.83	112.41	103.77	113.2	106.79	109.09	105.94	104.86	103.81
02/02/2017	110.52	105.87	112.31	103.79	113.2	106.8	109.31	105.94	104.91	103.76
03/02/2017	110.7	105.89	112.3	104.02	113.22	106.93	109.43	106.05	104.96	103.76
06/02/2017	111.25	105.92	112.32	104.35	113.21	107.02	109.72	106.21	105.11	104.23
07/02/2017	111.23	105.86	112.74	104.84	113.35	107.15	109.83	106.4	105.22	104.23
08/02/2017	111.24	105.9	112.74	104.8	113.32	107.14	110.21	106.39	105.21	104.52
09/02/2017	111.5	105.87	112.74	105.02	113.35	107.34	110.06	106.44	105.26	104.76
10/02/2017	111.37	105.8	112.54	104.94	113.25	107.43	109.96	106.39	105.15	104.63
13/02/2017	111.4	105.83	112.74	104.87	113.33	107.66	109.95	106.44	105.21	104.63

14/02/2017	111.36	105.82	112.31	104.87	113.35	107.73	109.89	106.4	105.21	104.42
15/02/2017	111.33	105.76	112.41	104.71	113.27	107.63	109.85	106.38	105.21	104.42
16/02/2017	111.32	105.74	112.22	104.77	113.24	107.65	109.88	106.45	105.15	104.09
17/02/2017	111.45	105.77	112.41	105.13	113.18	107.89	110.29	106.52	105.15	104.54
20/02/2017	111.58	105.78	112.41	105.21	113.38	108.43	110.29	106.54	105.16	104.54
21/02/2017	111.8	105.77	112.42	105.36	113.51	108.61	110.4	106.71	105.29	104.6
22/02/2017	111.63	105.83	112.46	105.52	113.43	109.04	110.53	106.76	105.39	104.54
23/02/2017	111.93	105.74	112.7	105.61	113.46	109.07	110.82	106.76	105.41	104.55
27/02/2017	112.35	105.59	112.74	106.08	113.46	109.28	111.33	107.01	105.52	105.23
28/02/2017	112.26	105.66	112.69	106.07	113.42	109.53	111.33	106.92	105.47	105.23
01/03/2017	112	105.54	112.49	105.77	113.35	109.32	110.67	106.66	105.3	105.4
02/03/2017	111.71	105.51	112.39	105.3	113.12	109.27	110.51	106.45	105.15	105.19
03/03/2017	111.61	105.49	112.34	105.26	113.1	109.24	110.59	106.4	105.15	104.89
06/03/2017	111.35	105.49	112.09	104.96	112.98	109.08	110.22	106.23	105	104.67
07/03/2017	111.27	105.54	112.24	105.02	113.06	108.93	110.42	106.35	105.1	104.35
08/03/2017	111.51	105.53	112.04	104.86	113.1	108.63	110.53	106.46	105.17	104.35
09/03/2017	111.18	105.55	111.94	104.57	113.04	107.98	110.06	106.26	105.07	104.01
10/03/2017	110.84	105.53	111.97	104.16	113.01	107.8	109.62	106.19	105.07	104.01

Annex 3: Covariance matrix

	Toyota Motor	**Renault**	**Nokia**	**Vodafone Group**	**The Royal Bank of Scotland**	**SoftBank**	**Sydney Airport**	**Pepsi**	**Philip Morris**	**Berlin, Land**
Toyota Motor	0.000003	0.000000	-0.000000	0.000003	0.000001	0.000000	0.000001	0.000001	0.000001	0.000001
Renault	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000000	0.000000	0.000000	0.000000
Nokia	-0.000000	0.000000	0.000003	0.000002	0.000000	0.000004	0.000001	0.000001	0.000001	0.000001
Vodafone Group	0.000003	0.000000	0.000002	0.000009	0.000001	0.000006	0.000004	0.000005	0.000004	0.000003
The Royal Bank of Scotland	0.000001	0.000000	0.000000	0.000001	0.000001	0.000001	0.000001	0.000000	0.000000	0.000000
SoftBank	0.000000	0.000001	0.000004	0.000006	0.000001	0.000020	0.000004	0.000005	0.000005	0.000003
Sydney Airport	0.000001	0.000000	0.000001	0.000004	0.000001	0.000004	0.000006	0.000002	0.000002	0.000002
Pepsi	0.000001	0.000000	0.000001	0.000005	0.000000	0.000005	0.000002	0.000004	0.000004	0.000002
Philip Morris	0.000001	0.000000	0.000001	0.000004	0.000000	0.000005	0.000002	0.000004	0.000004	0.000001
Berlin, Land	0.000001	0.000000	0.000001	0.000003	0.000000	0.000003	0.000002	0.000002	0.000001	0.000007

Annex 4: Yield curves derived from the annual transition matrix

2017

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	72.38%	10.76%	3.54%	1.72%	0.60%	0.81%	0.11%	0.07%	0.35%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA+	1.78%	56.30%	23.27%	5.43%	2.23%	1.63%	1.10%	0.92%	0.10%	0.42%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.38%	1.87%	62.98%	13.36%	7.45%	2.73%	1.22%	0.45%	0.07%	0.19%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA-	0.14%	0.19%	7.12%	57.31%	18.59%	5.54%	2.54%	0.49%	0.24%	0.28%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A+	0.01%	0.14%	1.25%	8.35%	55.96%	16.41%	5.20%	1.22%	0.72%	0.18%	0.02%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
A	0.01%	0.35%	0.38%	1.40%	7.13%	55.34%	17.08%	4.37%	2.03%	0.46%	0.14%	0.13%	0.11%	0.09%	0.01%	0.00%	0.01%	0.20%
A-	0.08%	0.02%	0.28%	0.37%	1.86%	10.99%	59.37%	12.16%	3.01%	1.42%	0.33%	0.18%	0.18%	0.04%	0.02%	0.00%	0.02%	0.11%
BBB+	0.00%	0.00%	0.03%	0.24%	0.58%	1.89%	11.31%	55.34%	12.73%	2.51%	0.78%	0.93%	0.37%	0.34%	0.16%	0.04%	0.16%	0.20%
BBB	0.00%	0.00%	0.17%	0.04%	0.35%	1.04%	1.99%	11.53%	50.40%	11.15%	2.80%	1.76%	0.33%	0.29%	0.52%	0.17%	0.18%	0.27%
BBB-	0.00%	0.00%	0.25%	0.03%	0.29%	1.02%	0.98%	2.82%	12.41%	46.76%	8.68%	4.06%	1.37%	0.74%	0.51%	0.08%	0.37%	0.71%
BB+	0.00%	0.00%	0.02%	0.00%	0.03%	0.11%	0.53%	1.10%	2.26%	14.90%	36.28%	9.91%	3.58%	2.64%	0.70%	0.13%	0.40%	0.27%
BB	0.00%	0.00%	0.00%	0.00%	0.01%	0.04%	0.41%	0.77%	0.32%	3.51%	13.14%	33.80%	7.78%	4.20%	1.58%	0.29%	0.75%	1.26%
BB-	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.03%	0.10%	0.35%	0.54%	1.36%	10.71%	36.58%	15.36%	5.60%	1.40%	0.72%	2.95%
B+	0.00%	0.00%	0.00%	0.00%	0.01%	0.30%	0.05%	0.32%	0.07%	0.13%	0.65%	4.10%	10.32%	32.13%	9.06%	4.85%	2.11%	4.19%
B	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.00%	0.04%	0.09%	0.48%	0.50%	0.92%	2.68%	12.48%	22.61%	8.78%	4.70%	8.85%
B-	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.05%	0.75%	0.07%	0.04%	0.07%	0.21%	2.27%	6.67%	26.40%	12.12%	21.42%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.06%	0.01%	0.02%	0.05%	0.15%	1.45%	3.13%	7.20%	8.02%	48.62%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

2018

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	61.7%	13.0%	5.6%	2.6%	1.1%	1.2%	0.3%	0.2%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA+	2.2%	42.6%	27.1%	7.7%	3.8%	2.5%	1.7%	1.2%	0.2%	0.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.5%	2.2%	50.7%	15.9%	9.9%	4.1%	2.0%	0.7%	0.2%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA-	0.2%	0.3%	8.4%	44.7%	21.4%	7.9%	3.8%	0.9%	0.4%	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
A+	0.0%	0.2%	1.8%	9.6%	43.3%	18.8%	7.5%	2.0%	1.1%	0.3%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
A	0.0%	0.4%	0.6%	1.9%	8.2%	42.8%	19.8%	6.0%	2.8%	0.8%	0.2%	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%	0.3%
A-	0.1%	0.1%	0.4%	0.6%	2.6%	12.7%	47.4%	14.2%	4.3%	1.9%	0.5%	0.3%	0.2%	0.1%	0.0%	0.0%	0.0%	0.2%
BBB+	0.0%	0.0%	0.1%	0.3%	0.8%	2.8%	13.1%	42.7%	14.2%	3.6%	1.2%	1.2%	0.5%	0.4%	0.2%	0.1%	0.2%	0.4%
BBB	0.0%	0.0%	0.2%	0.1%	0.5%	1.4%	3.0%	12.8%	37.3%	12.0%	3.5%	2.2%	0.6%	0.5%	0.6%	0.2%	0.2%	0.5%
BBB-	0.0%	0.0%	0.3%	0.1%	0.4%	1.3%	1.4%	3.9%	13.2%	33.6%	8.8%	4.6%	1.7%	1.1%	0.6%	0.2%	0.4%	1.1%
BB+	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.7%	1.5%	3.3%	14.6%	23.5%	9.4%	4.0%	3.0%	1.0%	0.3%	0.4%	0.7%
BB	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.5%	0.9%	0.7%	4.5%	11.8%	21.2%	7.4%	4.7%	1.8%	0.5%	0.7%	2.0%
BB-	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.4%	0.8%	2.2%	9.9%	23.7%	14.1%	5.6%	2.0%	1.0%	4.5%
B+	0.0%	0.0%	0.0%	0.0%	0.0%	0.3%	0.1%	0.3%	0.2%	0.3%	1.0%	4.3%	9.3%	20.2%	7.7%	4.6%	2.2%	6.6%
B	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.1%	0.2%	0.5%	0.5%	1.3%	3.1%	10.1%	12.2%	7.1%	3.8%	12.7%
B-	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.7%	0.1%	0.1%	0.2%	0.4%	2.5%	5.3%	14.8%	7.6%	29.0%
CCC	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.1%	0.3%	1.4%	2.3%	4.6%	3.3%	52.6%
D	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

⋮

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2026

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	17.63%	9.04%	11.12%	6.44%	5.11%	3.78%	2.54%	1.30%	0.81%	0.44%	0.12%	0.08%	0.04%	0.03%	0.02%	0.01%	0.01%	0.07%
AA+	1.75%	5.76%	15.60%	10.80%	9.89%	7.13%	5.31%	2.64%	1.36%	0.77%	0.23%	0.17%	0.09%	0.07%	0.04%	0.01%	0.02%	0.15%
AA	0.57%	1.43%	11.94%	11.17%	11.66%	8.85%	6.52%	2.94%	1.50%	0.74%	0.23%	0.16%	0.09%	0.07%	0.03%	0.01%	0.02%	0.16%
AA-	0.26%	0.61%	5.80%	10.61%	13.12%	11.34%	8.91%	4.11%	2.16%	1.00%	0.33%	0.24%	0.13%	0.10%	0.05%	0.02%	0.02%	0.25%
A+	0.11%	0.35%	2.61%	5.90%	10.61%	12.15%	10.96%	5.57%	2.99%	1.34%	0.46%	0.34%	0.19%	0.15%	0.08%	0.03%	0.04%	0.40%
A	0.09%	0.28%	1.25%	2.63%	5.76%	11.18%	13.30%	7.99%	4.48%	2.11%	0.78%	0.59%	0.33%	0.26%	0.14%	0.06%	0.06%	0.94%
A-	0.11%	0.16%	0.74%	1.49%	3.61%	8.63%	14.05%	9.99%	5.86%	2.91%	1.12%	0.84%	0.46%	0.37%	0.20%	0.09%	0.09%	0.99%
BBB+	0.04%	0.06%	0.37%	0.72%	1.79%	4.54%	8.95%	10.20%	7.42%	4.00%	1.66%	1.25%	0.67%	0.57%	0.32%	0.15%	0.14%	1.70%
BBB	0.02%	0.04%	0.28%	0.40%	0.98%	2.39%	4.64%	6.73%	7.14%	4.94%	2.22%	1.61%	0.88%	0.75%	0.41%	0.20%	0.17%	2.31%
BBB-	0.01%	0.03%	0.26%	0.30%	0.69%	1.58%	2.75%	4.09%	5.23%	5.08%	2.60%	1.94%	1.18%	1.02%	0.52%	0.27%	0.21%	3.54%
BB+	0.00%	0.01%	0.10%	0.11%	0.28%	0.73%	1.33%	2.07%	2.89%	3.59%	2.33%	1.94%	1.37%	1.25%	0.61%	0.34%	0.23%	3.66%
BB	0.00%	0.01%	0.05%	0.05%	0.14%	0.39%	0.73%	1.10%	1.48%	2.16%	1.76%	1.74%	1.45%	1.39%	0.68%	0.41%	0.25%	5.96%
BB-	0.00%	0.00%	0.02%	0.02%	0.06%	0.18%	0.30%	0.47%	0.64%	1.05%	1.15%	1.61%	1.86%	1.99%	1.03%	0.71%	0.39%	12.04%
B+	0.00%	0.00%	0.01%	0.02%	0.07%	0.18%	0.26%	0.34%	0.39%	0.59%	0.66%	1.01%	1.26%	1.47%	0.80%	0.62%	0.33%	15.34%
B	0.00%	0.00%	0.01%	0.01%	0.02%	0.08%	0.11%	0.18%	0.23%	0.32%	0.34%	0.53%	0.69%	0.89%	0.53%	0.46%	0.25%	22.09%
B-	0.00%	0.00%	0.00%	0.01%	0.02%	0.05%	0.08%	0.15%	0.19%	0.16%	0.13%	0.20%	0.27%	0.41%	0.29%	0.32%	0.17%	40.74%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.05%	0.05%	0.05%	0.08%	0.12%	0.17%	0.12%	0.12%	0.07%	56.98%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Annex 5: Forward yield curves from 2017 to 2026

	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
AAA	-0.20%	-0.02%	0.30%	0.55%	0.83%	1.05%	1.28%	1.51%	1.73%	1.86%
AA+	-0.20%	-0.02%	0.30%	0.55%	0.83%	1.05%	1.29%	1.51%	1.73%	1.86%
AA	-0.20%	-0.02%	0.30%	0.55%	0.83%	1.05%	1.29%	1.51%	1.73%	1.86%
AA-	-0.20%	-0.02%	0.30%	0.55%	0.83%	1.06%	1.29%	1.51%	1.74%	1.87%
A+	-0.19%	-0.01%	0.31%	0.56%	0.84%	1.06%	1.30%	1.52%	1.75%	1.87%
A	-0.10%	0.05%	0.36%	0.60%	0.88%	1.10%	1.33%	1.55%	1.78%	1.90%
A-	-0.15%	0.02%	0.34%	0.59%	0.87%	1.09%	1.33%	1.55%	1.78%	1.90%
BBB+	-0.10%	0.07%	0.39%	0.63%	0.92%	1.14%	1.37%	1.59%	1.82%	1.94%
BBB	-0.07%	0.10%	0.42%	0.67%	0.95%	1.17%	1.41%	1.63%	1.85%	1.98%
BBB-	0.15%	0.25%	0.54%	0.78%	1.05%	1.26%	1.49%	1.70%	1.92%	2.04%
BB+	-0.07%	0.14%	0.48%	0.74%	1.03%	1.25%	1.49%	1.71%	1.93%	2.05%
BB	0.42%	0.48%	0.75%	0.98%	1.24%	1.44%	1.65%	1.86%	2.07%	2.18%
BB-	1.28%	1.12%	1.33%	1.51%	1.74%	1.91%	2.10%	2.27%	2.45%	2.54%
B+	1.93%	1.70%	1.85%	1.97%	2.14%	2.26%	2.40%	2.54%	2.69%	2.75%
B	4.53%	3.48%	3.24%	3.10%	3.08%	3.05%	3.08%	3.13%	3.21%	3.21%
B-	13.09%	9.51%	7.99%	6.97%	6.31%	5.80%	5.46%	5.22%	5.07%	4.89%
CCC	45.95%	24.12%	16.90%	13.23%	11.10%	9.67%	8.71%	8.02%	7.54%	7.09%

Annex 6: Random variables

	Toyota Motor	Renault	Nokia	Vodafone Group	The Royal Bank of Scotland	SoftBank	Sydney Airport	Pepsi	Philip Morris	Berlin, Land
1	1.030	1.555	2.235	0.316	0.379	-0.078	0.674	1.334	-0.777	-1.640
2	-0.244	1.292	1.452	-0.775	1.725	-0.709	-0.336	-0.971	-0.260	0.823
3	1.533	-0.960	-2.178	0.882	-1.620	-1.924	0.622	-1.662	-0.415	0.396
4	1.743	-0.367	0.081	-0.052	2.342	0.204	-0.267	0.948	0.021	-0.851
5	-0.192	1.445	-0.856	0.249	0.333	0.468	3.107	-0.200	-0.946	1.357
6	-1.435	0.966	-1.171	-0.928	1.682	-0.910	-1.050	0.970	1.512	1.311
7	-0.255	0.088	-0.604	-1.150	-0.343	0.724	-0.992	0.885	2.425	0.292
8	-1.635	1.566	1.006	-0.202	-0.380	0.104	0.205	-0.149	-2.097	-1.764
9	0.150	-0.738	-1.445	0.879	0.793	1.011	0.267	-0.398	-1.521	0.142
10	-0.720	1.453	0.191	-0.730	0.352	-0.724	0.506	-2.124	-1.319	0.771
11	-0.513	-0.030	0.383	0.191	-0.200	1.611	1.187	1.654	-0.334	2.079
12	0.936	1.345	-0.447	2.006	-1.496	1.690	-0.002	0.583	0.174	0.687
13	0.534	0.668	-0.759	0.251	0.417	0.300	0.501	-2.089	0.724	0.621
14	0.105	-0.057	0.252	1.215	-0.737	0.167	-2.342	-0.843	1.669	0.197
15	2.356	0.156	1.217	1.202	-0.915	-0.541	1.201	-1.968	-0.727	-1.095
16	-1.273	1.219	1.122	-0.692	0.370	1.547	1.066	-0.406	0.617	1.000
17	0.638	0.505	-0.219	-0.177	0.138	-1.791	-0.307	-0.226	-0.338	-1.012
18	1.202	0.539	1.029	-0.754	-0.441	1.006	0.602	0.258	0.036	-0.494
19	-0.337	-0.820	-1.201	-0.636	-0.605	-0.987	-0.262	-0.014	0.728	0.988
20	0.546	-1.289	0.453	1.755	0.750	0.504	-0.667	1.535	-0.422	0.351
21	-0.999	-1.465	-0.997	-1.054	0.677	0.824	1.473	0.509	-0.044	0.259
22	-1.294	0.345	0.491	0.924	-0.924	-0.204	0.080	1.271	0.097	-0.400

23	-0.245	-0.467	-0.747	-0.779	0.109	1.161	1.006	1.052	-0.913	-0.319
24	3.023	0.863	1.685	-0.037	-0.228	-0.688	1.362	1.154	-1.157	-0.297
25	1.129	-0.586	1.512	-0.387	-0.291	0.412	0.037	1.514	-0.955	-1.117
26	-0.573	-1.280	-0.898	0.785	0.140	0.405	-0.771	-1.491	2.939	-1.875
27	-0.725	1.370	1.396	0.020	-0.739	-0.782	-0.181	1.177	1.720	1.051
28	-2.685	1.189	-0.776	1.268	-0.050	-0.417	-1.570	-0.932	-2.449	-0.212
29	0.297	0.931	3.297	1.301	1.517	-2.588	0.285	1.315	-0.035	-0.155
30	0.752	1.141	0.566	1.302	-0.509	-0.345	-0.333	-0.340	-0.988	-0.107
31	1.342	1.254	-0.473	-0.848	0.712	-2.676	-0.397	-2.112	0.554	0.272
32	-0.869	0.001	-0.810	-0.788	-0.795	-0.184	1.206	0.105	1.934	-0.039
ETC	…	…	…	…	…	…	…	…	…	…

Annex 7: Correlated random variables

	Toyota Motor	**Renault**	**Nokia**	**Vodafone Group**	**The Royal Bank of Scotland**	**SoftBank**	**Sydney Airport**	**Pepsi**	**Philip Morris**	**Berlin, Land**
1	1.4079121	2.3986378	2.2545939	0.535158	0.6052565	-0.036987	0.4620451	0.2201214	-0.086338	-1.47838
2	-0.226836	1.1358079	0.7603677	-1.305528	1.3122601	-0.729342	-0.264816	-0.636187	-0.138777	0.7424907
3	1.1649439	-2.471367	-3.482542	-1.028338	-1.784071	-1.686008	0.4492113	-1.153015	-0.14808	0.3569828
4	2.4194825	0.2640704	0.5769632	0.6218039	2.2698447	0.1736887	-0.23762	0.4918032	0.0742267	-0.767367
5	1.3176389	1.964642	0.0099011	0.828885	0.4498788	0.8647513	2.5920756	-0.520035	-0.373827	1.2232705
6	-0.823902	1.0532367	-0.598505	0.5772033	1.2468754	-0.383948	-0.667127	1.4817302	0.3187023	1.1818135
7	-0.566829	0.5516891	0.4812528	0.9734624	-0.227343	0.970398	-0.667635	1.8460784	0.656155	0.2637107
8	-2.557865	0.99686	-0.288583	-1.695937	-0.186164	-0.432033	-0.038619	-1.360117	-0.446483	-1.590664
9	0.6429462	-0.912921	-1.258178	0.0291756	0.7928874	0.5401807	0.1594996	-1.0416	-0.437687	0.1277875
10	-1.333019	0.3361081	-1.301172	-2.484753	0.0674125	-0.945477	0.3311285	-1.879572	-0.431612	0.695195
11	0.8865999	1.2881841	2.2667685	2.0843329	-0.075004	1.8651806	1.1812502	0.938063	-0.260164	1.8740974
12	1.9755537	2.216106	1.1042187	2.4224197	-1.203567	1.5163326	0.0802047	0.485226	-0.006242	0.6194297
13	0.7400982	0.4777669	-0.692227	-0.273945	0.3575468	0.2103192	0.3864869	-0.76476	0.1532442	0.5594686
14	-0.177315	-0.062039	0.4537722	0.6993764	-0.854496	-0.103028	-1.87674	0.4324337	0.4521401	0.1773311
15	2.1297066	-0.135611	0.2115805	-0.769652	-0.748109	-0.72543	0.7882953	-1.61627	-0.116039	-0.987442
16	-0.805964	2.1571329	2.1805352	0.5340264	0.5299579	1.5588174	0.9453684	0.1783935	0.0928645	0.9012602
17	0.0896126	-0.355923	-1.536422	-1.21717	-0.034206	-1.65969	-0.345534	-0.393262	-0.01368	-0.912817
18	0.8530483	0.9810836	1.3150739	-0.070932	-0.146152	0.9018458	0.4674413	0.1293614	0.0496927	-0.445034
19	-0.645481	-1.423401	-1.43957	-0.3266	-0.795919	-0.61099	-0.117286	0.4637057	0.1248111	0.8906387
20	1.8681702	-0.292331	1.5661743	2.1110531	0.6753642	0.4605876	-0.46441	0.6855936	-0.146311	0.3168195
21	-0.69852	-1.117176	-0.269428	0.2516303	0.8452651	1.0077469	1.2411762	0.2905757	-0.032986	0.233189
22	-0.832143	0.8607952	0.9482524	1.3305122	-0.773019	0.0159242	0.0949934	0.7547986	0.0593284	-0.360492

23	-0.226638	-0.237651	-0.248834	0.0935033	0.3799336	1.0639457	0.8123484	0.0882024	-0.230282	-0.287199
24	3.4370717	1.2956908	1.4821193	0.1330956	-0.144474	-0.361471	1.1009887	0.0161721	-0.300587	-0.267601
25	0.7676003	-0.175466	1.4423182	-0.022534	-0.06515	0.2810222	-0.017034	0.2689424	-0.178145	-1.006895
26	-0.848312	-1.003336	-0.377248	0.9691848	0.4018163	0.2994605	-0.733226	0.5819946	0.9739142	-1.690171
27	-0.106029	2.2012735	2.1959203	1.5394365	-0.818787	-0.084565	0.0387278	1.6932714	0.3979448	0.9472172
28	-3.18875	-0.15145	-2.281551	-1.611172	-0.342096	-1.191776	-1.418442	-1.879657	-0.669266	-0.191348
29	1.837684	1.9652688	3.1110647	1.25129	1.1292466	-1.828069	0.2780665	0.7279776	0.0027615	-0.139992
30	0.9280811	0.9421784	0.090109	-0.040966	-0.57502	-0.563486	-0.32751	-0.739256	-0.268451	-0.09616
31	0.6014378	-0.210645	-2.359356	-2.355249	0.1894712	-2.373213	-0.378325	-0.898029	0.1334737	0.2451205
32	-0.823141	0.3045186	-0.139434	0.7789192	-0.555202	0.4181902	1.0502922	1.1042031	0.5452086	-0.035151
ETC	…	…	…	…	…	…	…	…	…	…

Annex 8: Breakpoints

Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC
AAA	1.65	1.84	1.67	1.79			x										
AA+	-1.28	1.71	1.64	1.78	1.60	1.59											
AA	-1.84	-0.81	1.54	1.77	1.60	1.57	1.67		x	x							
AA-	-2.11	-1.58	-1.04	1.38	1.54	1.56	1.65	1.54									
A+	-2.38	-1.88	-1.53	-0.97	1.19	1.50	1.64	1.52	1.34	1.29							
A	-2.48	-2.02	-2.02	-1.71	-1.06	1.21	1.55	1.50	1.33	1.28				0.93			
A-		-2.21	-2.38	-2.13	-1.81	-1.03	1.11	1.44	1.30	1.25	1.04	0.91					
BBB+		-2.40	-2.72	-2.64	-2.36	-1.76	-1.25	1.04	1.25	1.22	1.03	0.90		0.92			
BBB	x			-2.81	-2.64	-2.13	-1.93	-1.20	0.90	1.15	1.00		1.17			x	
BBB-		x	x	-2.93	-3.19	-2.56	-2.26	-1.90	-1.20	0.79	0.97	0.88	1.16		0.75		
BB+						-2.70	-2.68	-2.14	-1.80	-1.21	0.57	0.82	1.15	0.91	0.74		
BB						-2.76	-2.86	-2.24	-2.06	-1.67	-1.13	0.47	1.13	0.90	0.73		
BB-						-2.83	-2.95	-2.46	-2.37	-2.04	-1.67	-1.21	0.78	0.80	0.71		0.74
B+						-2.93		-2.58	-2.41	-2.23	-1.98	-1.64	-0.87	0.52	0.67	0.95	0.74
B								-2.77	-2.46	-2.34	-2.51	-2.00	-1.52	-1.02	0.32	0.90	0.70
B-									-2.75				-2.01	-1.48	-0.95	0.68	0.60
CCC								-2.86	-2.88	-2.49	x	-2.24	-2.11	-1.86	-1.36	-0.64	0.34
D						x	x	-3.19	-3.09	-2.73		-2.58	-2.18	-2.09	-1.73	-1.27	-0.31

Annex 9: Rating assignments

	Toyota Motor	Renault	Nokia	Vodafone Group	The Royal Bank of Scotland	SoftBank	Sydney Airport	Pepsi	Philip Morris	Berlin, Land
Default	**AAA**	**BBB-**	**BB+**	**BBB+**	**A-**	**BB+**	**BBB**	**A+**	**AA**	**AA+**
1	AA+	AA-	A	AA	AA+	BB	AA-	AAA	AA-	AA-
2	AA+	AA-	A	BBB-	AA+	BB	BBB-	A	AA-	AA
3	AA+	CCC	B	BBB	BBB	B	AA-	A-	AA-	AA
4	AAA	AA-	A	AA	AA+	A	BBB-	AAA	AA	AA
5	AA+	AA-	A	AA	AA+	A	AA-	A	AA-	AA
6	AA+	AA-	BB	AA	AA+	BB	BBB-	AAA	AA	AA
7	AA+	AA-	A	AA	BBB+	A	BBB-	AAA	AA	AA
8	A	AA-	BB	BBB-	BBB+	BB	BBB-	A-	AA-	A+
9	AA+	BB+	BB-	AA	AA+	A	AA-	A	AA-	AA
10	AA	AA-	BB-	B+	AA+	BB	AA-	BBB+	AA-	AA
11	AA+	AA-	A	AA	BBB+	A	AA-	AAA	AA-	AAA
12	AAA	AA-	A	AA	BBB+	A	AA-	AAA	AA-	AA
13	AA+	AA-	BB	BBB	AA+	A	AA-	A	AA	AA
14	AA+	BB+	A	AA	BBB+	BB	BB	AAA	AA	AA
15	AAA	BB+	A	BBB	BBB+	BB	AA-	A-	AA-	AA-
16	AA+	AA-	A	AA	AA+	A	AA-	AAA	AA	AA
17	AA+	BB+	BB-	BBB-	BBB+	BB-	BBB-	A	AA-	AA-
18	AA+	AA-	A	BBB	BBB+	A	AA-	AAA	AA	AA
19	AA+	BB	BB-	BBB	BBB+	BB	BBB-	AAA	AA	AA
20	AAA	BB+	A	AA	AA+	A	BBB-	AAA	AA-	AA

21	AA+	BB+	BB	AA	AA+	A	AA-	AAA	AA-	AA
22	AA+	AA-	A	AA	BBB+	A	AA-	AAA	AA	AA
23	AA+	BB+	BB	AA	AA+	A	AA-	AAA	AA-	AA
24	AAA	AA-	A	AA	BBB+	BB	AA-	AAA	AA-	AA
25	AA+	BB+	A	BBB	BBB+	A	BBB-	AAA	AA-	AA-
26	AA+	BB+	BB	AA	AA+	A	BBB-	AAA	AA	A+
27	AA+	AA-	A	AA	BBB+	BB	AA-	AAA	AA	AA
28	A	BB+	B	BBB-	BBB+	BB-	BB+	BBB+	AA-	AA
29	AAA	AA-	A	AA	AA+	B	AA-	AAA	AA	AA
30	AA+	AA-	A	BBB	BBB+	BB	BBB-	A	AA-	AA
31	AA+	BB+	B	BB-	AA+	B	BBB-	A	AA	AA
32	AA+	AA-	BB	AA	BBB+	A	AA-	AAA	AA	AA
ETC	…	…	…	…	…	…	…	…	…	…

Annex 10: Values of bonds by rating and number of pieces (€)

	Toyota Motor	Renault	Nokia	Vodafone Group	The Royal Bank of Scotland	SoftBank	Sydney Airport	Pepsi	Philip Morris	Berlin, Land
	1000	1000	20	10	20	5	10	10	10	1000
1	1,075,583	1,072,915	1,191,101	1,269,262	1,196,727	1,334,160	1,097,677	1,045,847	1,047,687	948,219
2	1,075,583	1,072,915	1,191,101	1,255,146	1,196,727	1,334,160	1,082,560	1,043,275	1,047,687	948,628
3	1,075,583	697,462	1,097,656	1,261,190	1,191,406	1,215,866	1,097,677	1,043,690	1,047,687	948,628
4	1,075,814	1,072,915	1,191,101	1,269,262	1,196,727	1,363,989	1,082,560	1,045,847	1,047,792	948,628
5	1,075,583	1,072,915	1,191,101	1,269,262	1,196,727	1,363,989	1,097,677	1,043,275	1,047,687	948,628
6	1,075,583	1,072,915	1,177,781	1,269,262	1,196,727	1,334,160	1,082,560	1,045,847	1,047,792	948,628
7	1,075,583	1,072,915	1,191,101	1,269,262	1,193,017	1,363,989	1,082,560	1,045,847	1,047,792	948,628
8	1,072,448	1,072,915	1,177,781	1,255,146	1,193,017	1,334,160	1,082,560	1,043,690	1,047,687	947,501
9	1,075,583	1,069,548	1,158,820	1,269,262	1,196,727	1,363,989	1,097,677	1,043,275	1,047,687	948,628
10	1,075,571	1,072,915	1,158,820	1,189,086	1,196,727	1,334,160	1,097,677	1,041,527	1,047,687	948,628
11	1,075,583	1,072,915	1,191,101	1,269,262	1,193,017	1,363,989	1,097,677	1,045,847	1,047,687	949,063
12	1,075,814	1,072,915	1,191,101	1,269,262	1,193,017	1,363,989	1,097,677	1,045,847	1,047,687	948,628
13	1,075,583	1,072,915	1,177,781	1,261,190	1,196,727	1,363,989	1,097,677	1,043,275	1,047,792	948,628
14	1,075,583	1,069,548	1,191,101	1,269,262	1,193,017	1,334,160	1,070,329	1,045,847	1,047,792	948,628
15	1,075,814	1,069,548	1,191,101	1,261,190	1,193,017	1,334,160	1,097,677	1,043,690	1,047,687	948,219
16	1,075,583	1,072,915	1,191,101	1,269,262	1,196,727	1,363,989	1,097,677	1,045,847	1,047,792	948,628
17	1,075,583	1,069,548	1,158,820	1,255,146	1,193,017	1,294,874	1,082,560	1,043,275	1,047,687	948,219
18	1,075,583	1,072,915	1,191,101	1,261,190	1,193,017	1,363,989	1,097,677	1,045,847	1,047,792	948,628
19	1,075,583	1,062,467	1,158,820	1,261,190	1,193,017	1,334,160	1,082,560	1,045,847	1,047,792	948,628
20	1,075,814	1,069,548	1,191,101	1,269,262	1,196,727	1,363,989	1,082,560	1,045,847	1,047,687	948,628
21	1,075,583	1,069,548	1,177,781	1,269,262	1,196,727	1,363,989	1,097,677	1,045,847	1,047,687	948,628
22	1,075,583	1,072,915	1,191,101	1,269,262	1,193,017	1,363,989	1,097,677	1,045,847	1,047,792	948,628

23	1,075,583	1,069,548	1,177,781	1,269,262	1,196,727	1,363,989	1,097,677	1,045,847	1,047,687	948,628
24	1,075,814	1,072,915	1,191,101	1,269,262	1,193,017	1,334,160	1,097,677	1,045,847	1,047,687	948,628
25	1,075,583	1,069,548	1,191,101	1,261,190	1,193,017	1,363,989	1,082,560	1,045,847	1,047,687	948,219
26	1,075,583	1,069,548	1,177,781	1,269,262	1,196,727	1,363,989	1,082,560	1,045,847	1,047,792	947,501
27	1,075,583	1,072,915	1,191,101	1,269,262	1,193,017	1,334,160	1,097,677	1,045,847	1,047,792	948,628
28	1,072,448	1,069,548	1,097,656	1,255,146	1,193,017	1,294,874	1,082,464	1,041,527	1,047,687	948,628
29	1,075,814	1,072,915	1,191,101	1,269,262	1,196,727	1,215,866	1,097,677	1,045,847	1,047,792	948,628
30	1,075,583	1,072,915	1,191,101	1,261,190	1,193,017	1,334,160	1,082,560	1,043,275	1,047,687	948,628
31	1,075,583	1,069,548	1,097,656	1,212,041	1,196,727	1,215,866	1,082,560	1,043,275	1,047,792	948,628
32	1,075,583	1,072,915	1,177,781	1,269,262	1,193,017	1,363,989	1,097,677	1,045,847	1,047,792	948,628
33	1,072,448	1,062,467	1,177,781	1,255,146	1,193,017	1,334,160	1,082,464	1,043,275	1,047,792	948,628
34	1,075,583	1,072,915	1,177,781	1,269,262	1,193,017	1,334,160	1,097,677	1,043,275	1,047,687	948,628
35	1,075,814	1,072,915	1,191,101	1,269,262	1,193,017	1,363,989	1,097,677	1,045,847	1,047,792	948,628
36	1,075,583	1,072,915	1,191,101	1,269,262	1,193,017	1,363,989	1,082,560	1,045,847	1,047,792	948,628
ETC.	…	…	…	…	…	…	…	…	…	…

Annex 11: Probability distribution of the portfolio value

Scenario	Values (€)	Frequency	Cumulative frequency	R1	R2
1	9,227,883	1	1	0.004%	0.004%
2	9,301,843	0	1	0.000%	0.004%
3	9,375,804	0	1	0.000%	0.004%
4	9,449,764	0	1	0.000%	0.004%
5	9,523,724	0	1	0.000%	0.004%
6	9,597,685	0	1	0.000%	0.004%
7	9,671,645	0	1	0.000%	0.004%
8	9,745,605	17	18	0.068%	0.072%
9	9,819,566	12	30	0.048%	0.120%
10	9,893,526	21	51	0.084%	0.204%
11	9,967,486	6	57	0.024%	0.228%
12	10,041,447	7	64	0.028%	0.256%
13	10,115,407	2	66	0.008%	0.264%
14	10,189,368	6	72	0.024%	0.288%
15	10,263,328	8	80	0.032%	0.320%
16	10,337,288	34	114	0.136%	0.456%
17	10,411,249	80	194	0.320%	0.776%
18	10,485,209	95	289	0.380%	1.156%
19	10,559,169	86	375	0.344%	1.500%
20	10,633,130	143	518	0.572%	2.072%
21	10,707,090	200	718	0.800%	2.872%
22	10,781,051	78	796	0.312%	3.184%
23	10,855,011	42	838	0.168%	3.352%
24	10,928,971	81	919	0.324%	3.676%
25	11,002,932	66	985	0.264%	3.940%
26	11,076,892	262	1247	1.048%	4.988%
27	11,150,852	1144	2391	4.576%	9.564%
28	11,224,813	3230	5621	12.920%	22.484%
29	11,298,773	15308	20929	61.232%	83.716%
30	11,372,733	4071	25000	16.284%	100.000%